ELECTRICAL CIRCUITS

COURSE MATERIAL

B.TECH
EEE-50202
(I YEAR – I SEM)

Department of Electrical and Electronics Engineering

MALLA REDDY ENGINEERING COLLEGE

Autonomous

MODULE –I
INTRODUCTION TO ELECTRICAL CIRCUITS

INTRODUCTION:

An Electric circuit is an interconnection of various elements in which there is at least one closed path in which current can flow. An Electric circuit is used as a component for any engineering system.

The performance of any electrical device or machine is always studied by drawing its electrical equivalent circuit. By simulating an electric circuit, any type of system can be studied for e.g., mechanical, hydraulic thermal, nuclear, traffic flow, weather prediction etc.

All control systems are studied by representing them in the form of electric circuits. The analysis, of any system can be learnt by mastering the techniques of circuit theory.

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Elements of an Electric circuit:

An Electric circuit consists of following types of elements.

Active elements:

Active elements are the elements of a circuit which possess energy of their own and can impart it to other element of the circuit.

Active elements are of two types

a) Voltage source   b) Current source

A Voltage source has a specified voltage across its terminals, independent of current flowing through it.
A current source has a specified current through it independent of the voltage appearing across it.

Passive Elements:

The passive elements of an electric circuit do not possess energy of their own. They receive energy from the sources. The passive elements are the resistance, the inductance and the capacitance. When electrical energy is supplied to a circuit element, it will respond in one and more of the following ways.

If the energy is consumed, then the circuit element is a pure resistor.

If the energy is stored in a magnetic field, the element is a pure inductor.

And if the energy is stored in an electric field, the element is a pure capacitor.

Linear and Non-Linear Elements.

Linear elements show the linear characteristics of voltage & current. That is its voltage-current characteristics are at all-times a straight-line through the origin.

For example, the current passing through a resistor is proportional to the voltage applied through its and the relation is expressed as V or $V = IR$. A linear element or network is one which satisfies the principle of superposition, i.e., the principle of homogeneity and additivity.

Resistors, inductors and capacitors are the examples of the linear elements and their properties do not change with a change in the applied voltage and the circuit current.

Non linear element's V-I characteristics do not follow the linear pattern i.e. the current passing through it does not change linearly with the linear change in the voltage across it. Examples are the semiconductor devices such as diode, transistor.

Bilateral and Unilateral Elements:

An element is said to be bilateral, when the same relation exists between voltage and current for the current flowing in both directions.

Ex: Voltage source, Current source, resistance, inductance & capacitance.

The circuits containing them are called bilateral circuits.

An element is said to be unilateral, when the same relation does not exist between voltage and current when current flowing in both directions. The circuits containing them are called unilateral circuits.

Ex: Vacuum diodes, Silicon Diodes, Selenium Rectifiers etc.

Lumped and Distributed Elements

Lumped elements are those elements which are very small in size & in which simultaneous actions takes place. Typical lumped elements are capacitors, resistors, inductors.

Distributed elements are those which are not electrically separable for analytical purposes.

For example a transmission line has distributed parameters along its length and may extend for hundreds of miles.

The circuits containing them are called unilateral circuits.

Independent & Dependent sources:

If the voltage of the voltage source is completely independent source of current and the current of the current source is completely independent of the voltage, then the sources are called as independent sources.

The special kind of sources in which the source voltage or current depends on some other quantity in the circuit which may be either a voltage or a current anywhere in the circuit are called Dependent sources or Controlled sources.

There are four possible dependent sources:

a. Voltage dependent Voltage source
b. Current dependent Current source
c. Voltage dependent Current source
d. Current dependent Current source
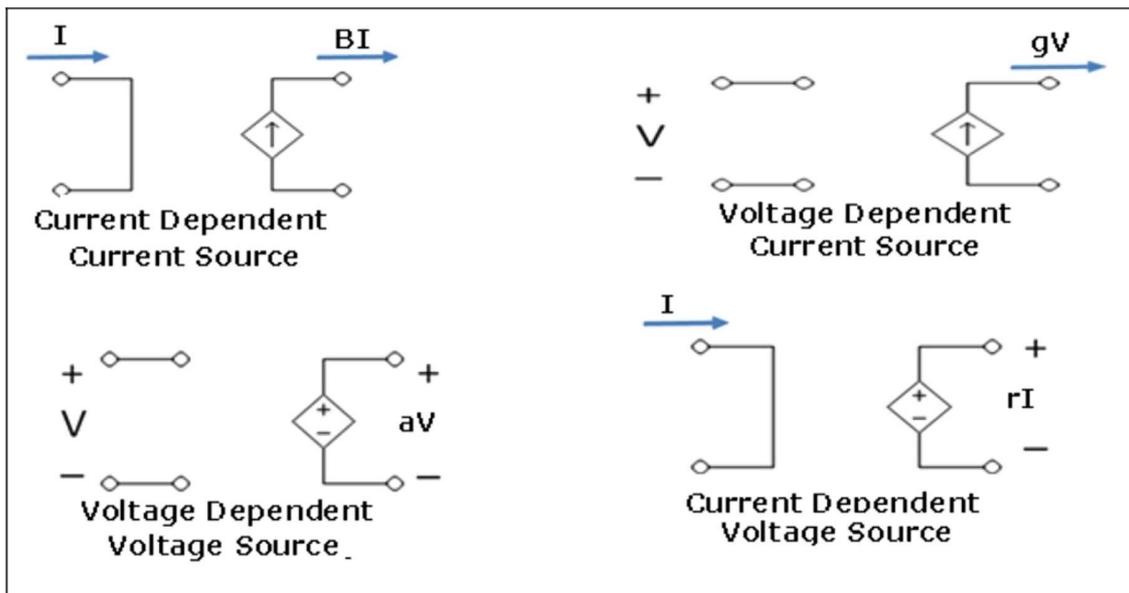
The constants of proportionalities are written as B, g, a, r in which B & a has no units, r has units of ohm & g units of mhos.

Independent sources actually exist as physical entities such as battery, a dc generator & an alternator. But dependent sources are used to represent electrical properties of electronic devices such as OPAMPS & Transistors.

Ideal & Practical sources:

1. An ideal voltage source is one which delivers energy to the load at a constant terminal voltage, irrespective of the current drawn by the load.

2. An ideal current source is one, which delivers energy with a constant current to the load, irrespective of the terminal voltage across the load.

3. A Practical voltage source always possesses a very small value of internal resistance r. The internal resistance of a voltage source is always connected in series with it & for a current source; it is always connected in parallel with it. As the value of the internal resistance of a practical voltage source is very small, its terminal voltage is assumed to be almost constant within a certain limit of current flowing through the load.

4. A practical current source is also assumed to deliver a constant current, irrespective of the terminal voltage across the load connected to it.
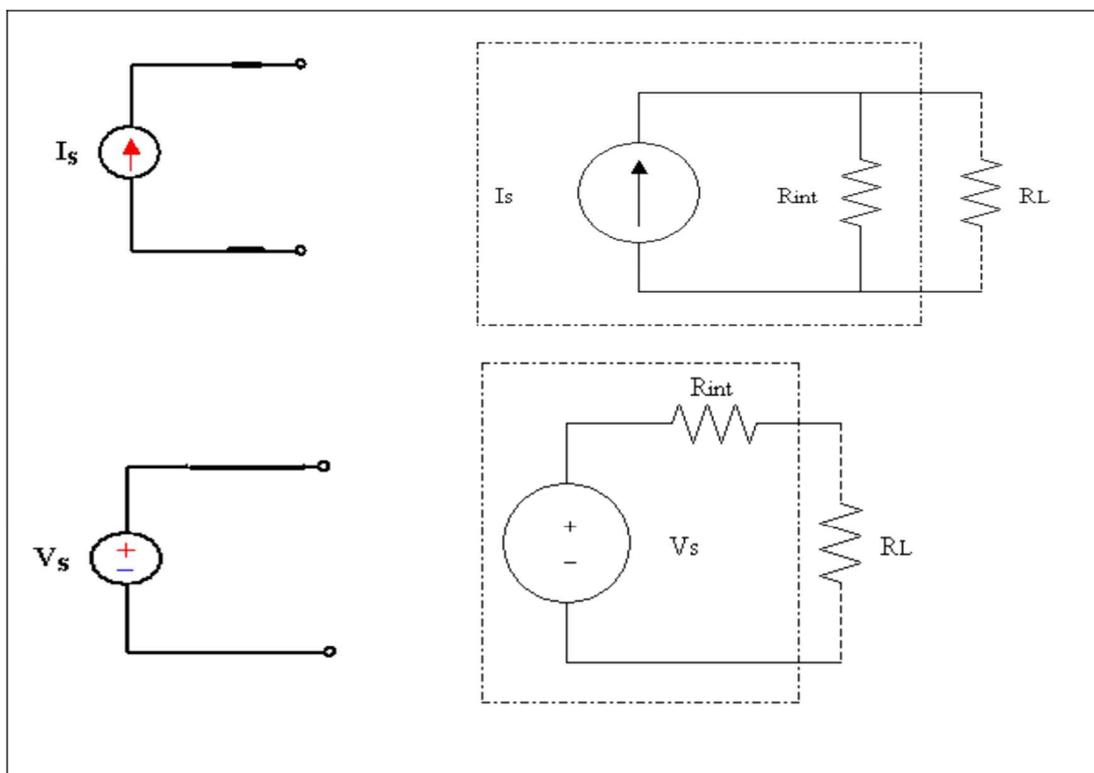
Ideal voltage source connected in series:
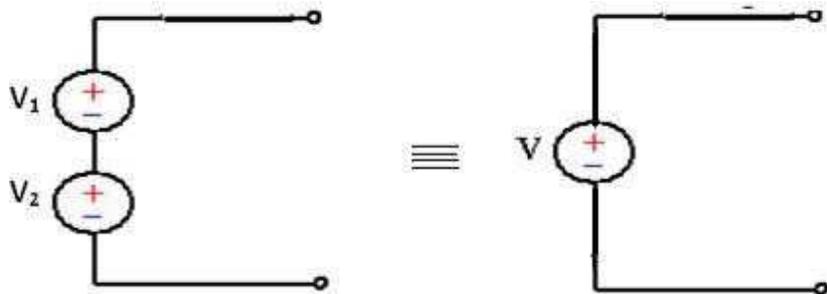
The equivalent single ideal voltage some is given by   V= V1 + V2

Any number of ideal voltage sources connected in series can be represented by a single ideal voltage some taking in to account the polarities connected together in to consideration.

Practical voltage source connected in series:
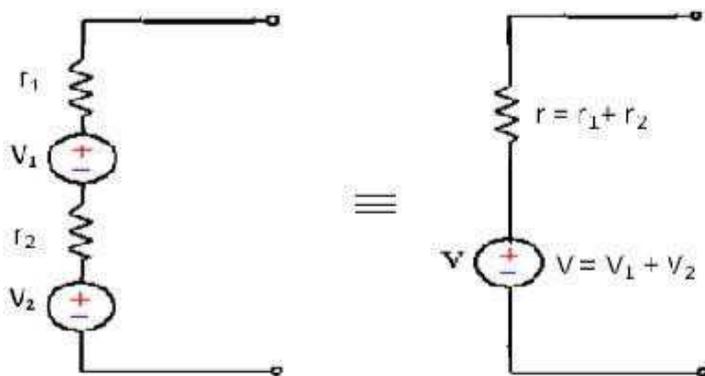
Ideal voltage source connected in parallel:
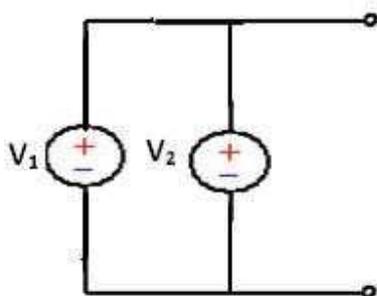
When two ideal voltage sources of emf's V1 & V2 are connected in parallel, what voltage appears across its terminals is ambiguous.

Hence such connections should not be made.

However if V1 = V2= V, then the equivalent voltage some is represented by V.

In that case also, such a connection is unnecessary as only one voltage source serves the purpose.

Practical voltage sources connected in parallel:

Equivalent Circuit   Single Equivalent Voltage Source
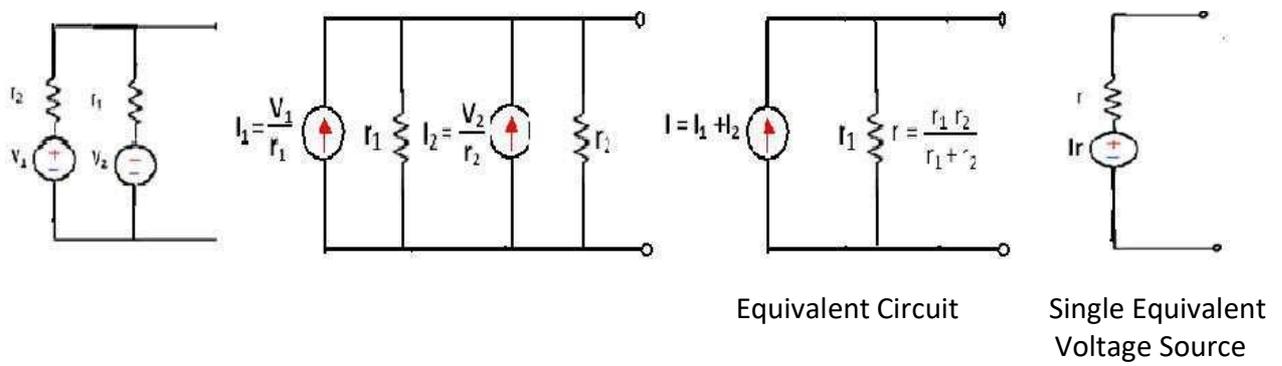
Ideal current sources connected in series:
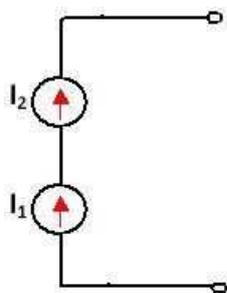
When ideal current sources are connected in series, what current flows through the line is ambiguous. Hence such a connection is not permissible.

However, it I1 = I2 = I, then the current in the line is I.

But, such a connection is not necessary as only one current source serves the purpose.

Practical current sources connected in series:
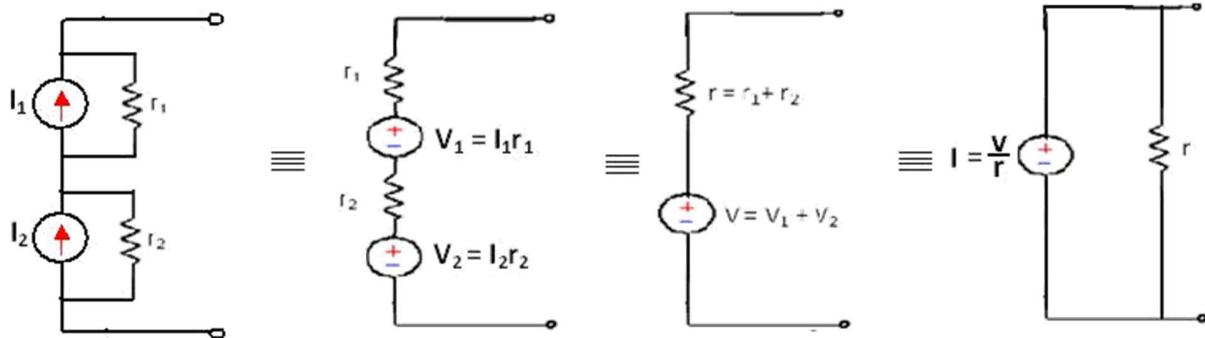
Ideal current sources connected in parallel
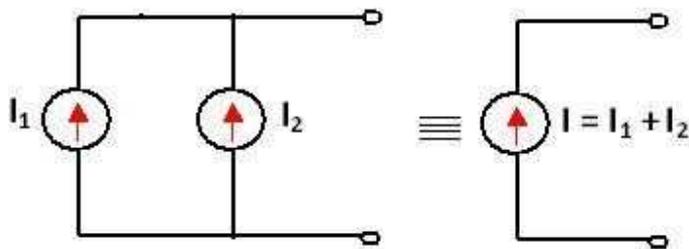
Two ideal current sources in parallel can be replaced by a single equivalent ideal current source.

Practical current sources connected in parallel
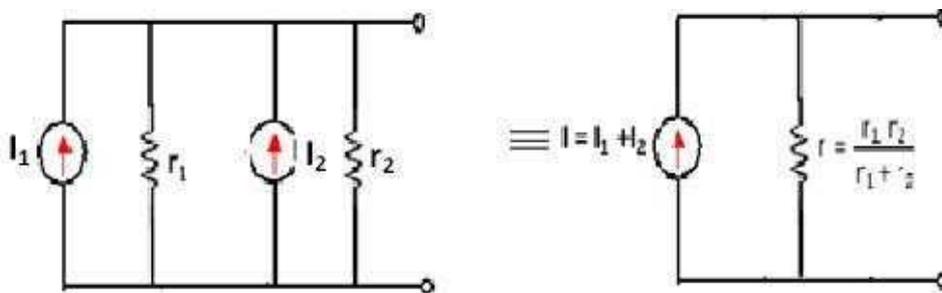
Source transformation:

A current source or a voltage source drives current through its load resistance and the magnitude of the current depends on the value of the load resistance.

Consider a practical voltage source and a practical current source connected to the same load resistance RL as shown in the figure
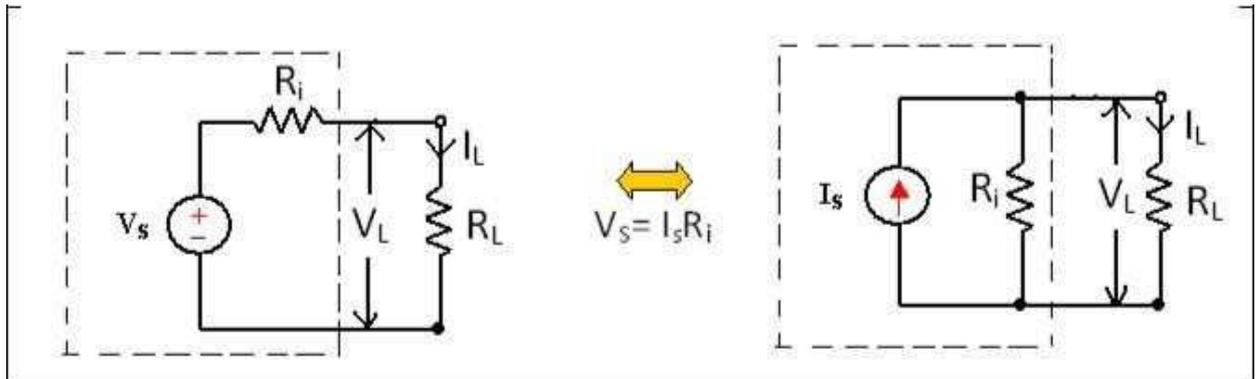
R1's in figure represents the internal resistance of the voltage source VS and current source IS.

Two sources are said to be identical, when they produce identical terminal voltage VL and load current IL.

The circuit in figure represents a practical voltage source & a practical current source respectively, with load connected to both the sources.

The terminal voltage VL and load current IL across their terminals are same.

Hence the practical voltage source & practical current source shown in the dotted box of figure are equal.

The two equivalent sources should also provide the same open circuit voltage & short circuit current.

From fig (a)

$$IL = \frac{Vs}{R+R_L}$$

From fig (b)

$$IL = I\frac{r}{R+R_L}$$

$$\therefore \frac{Vs}{R+R_L} = I$$

$$VS = IR \quad \text{or } I = \frac{Vs}{R}$$

Hence a voltage source Vs in series with its internal resistance R can be converted into a current source I = $\frac{Vs}{R}$, with its internal resistance R connected in parallel with it. Similarly a current source I in parallel with its internal resistance R can be converted into a voltage source V = IR in series with its internal resistance R.

R-L-C Parameters:

1. *Resistance:*

Resistance is that property of a circuit element which opposes the flow of electric current and in doing so converts electrical energy into heat energy.

It is the proportionality factor in ohm's law relating voltage and current.

Ohm's law states that the voltage drop across a conductor of given length and area of cross section is directly proportional to the current flowing through it.

$$R \propto i$$

$$V = Ri$$

$$i = \frac{\overline{}}{R} = GV$$

Where the reciprocal of resistance is called conductance G. The unit of resistance is ohm and the unit of conductance is mho or Siemens.

When current flows through any resistive material, heat is generated by the collision of electrons with other atomic particles. The power absorbed by the resistor is converted to heat and is given by the expression

P= vi= i2R where i is the resistor in amps, and v is the voltage across the resistor in volts.

Energy lost in a resistance in time t is given by

$$W = \int_0^t p\, dt = pt = i^2 Rt = \frac{v2}{R}\, t$$

2. *Inductance:*

Inductance is the property of a material by virtue of which it opposes any change of magnitude and direction of electric current passing through conductor. A wire of certain length, when twisted into a coil becomes a basic conductor. A change in the magnitude of the current changes the electromagnetic field.

Increase in current expands the field & decrease in current reduces it. A change in current produces change in the electromagnetic field. This induces a voltage across the coil according to Faradays laws of Electromagnetic Induction.

Induced Voltage $V = L\dfrac{di}{dt}$

V = Voltage across inductor in volts

I = Current through inductor in amps

$di = \dfrac{1}{L} v\, dt$

Integrating both sides,

$$\int_0^t di = \frac{1}{L}\int_0^t v\, dt$$

Power absorbed by the inductor $P = VI = Li\,\dfrac{di}{dt}$

Energy stored by the inductor

$$W = \int_0^t P\, dt = \int_0^t Li\,\frac{di}{dt}\, dt = \frac{Li^2}{2}$$

$$W = \frac{Li^2}{2}$$

Conclusions:

1) $V = L\dfrac{di}{dt}$

> The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.

2) For minute change in current within zero time (dt = 0) gives an infinite voltage across the inductor which is physically not at all feasible. In an inductor, the current cannot change abruptly. An inductor behaves as open circuit just after switching across dc voltage.

3) The inductor can store finite amount of energy, even if the voltage across the inductor is zero.
4) A pure inductor never dissipates energy, it only stores it. Hence it is also called as a non–dissipative passive element. However, physical inductor dissipates power due to internal resistance.

Ex: The current in a 2H inductor raises at a rate of 2A/s .Find the voltage across the inductor the energy stored in the magnetic field at after 2sec.

Sol:

$$V = L \frac{di}{dt}$$

$$= 2 \times 2 = 4V$$

MODULE-II
NETWORK TOPOLOGY

An important step in the procedure for solving any circuit problem consists first in selecting a number of independent branch currents as (known as loop currents or mesh currents) variables, and then to express all branch currents as functions of the chosen set of branch currents. Alter- nately a number of independent node pair voltages may be selected as variables and then express all existing node pair voltages in terms of these selected variables.

For simple networks involving a few elements, there is no difficulty in selecting the inde- pendent branch currents or the independent node-pair voltages. The set of linearly independent equations can be written by inspection. However for large scale networks particularly modern electronic circuits such as integrated circuits and microcircuits with a larger number of inter- connected branches, it is almost impossible to write a set of linearly independent equations by inspection or by mere intuition. The problem becomes quite difficult and complex. A system- atic and step by step method is therefore required to deal with such networks. Network topology (graph theory approach) is used for this purpose. By this method, a set of linearly independent loop or node equations can be written in a form that is suitable for a computer solution.

2.1 Terms and definitions

The description of networks in terms of their geometry is referred to as network topology. The adequacy of a set of equations for analyzing a network is more easily determined topologically than algebraically.

Graph (or linear graph): A network graph is a network in which all nodes and loops are re- tained but its branches are represented by lines. The voltage sources are replaced by short circuits and current sources are replaced by open circuits. (Sources without internal impedances or ad- mittances can also be treated in the same way because they can be shifted to other branches by E-shift and/or I-shift operations.)

Branch: A line segment replacing one or more network elements that are connected in series or parallel.

Node: Interconnection of two or more branches. It is a terminal of a branch. Usually intercon- nections of three or more branches are nodes.

Path: A set of branches that may be traversed in an order without passing through the same node more than once.

Loop: Any closed contour selected in a graph.

Mesh: A loop which does not contain any other loop within it.

Planar graph: A graph which may be drawn on a plane surface in such a way that no branch passes over any other branch.

Non-planar graph: Any graph which is not planar.

Oriented graph: When a direction to each branch of a graph is assigned, the resulting graph is called an oriented graph or a directed graph.

Connected graph: A graph is connected if and only if there is a path between every pair of nodes.

Sub graph: Any subset of branches of the graph.

Tree: A connected sub-graph containing all nodes of a graph but no closed path. i.e. it is a set of branches of graph which contains no loop but connects every node to every other node not necessarily directly. A number of different trees can be drawn for a given graph.

Link: A branch of the graph which does not belong to the particular tree under consideration. The links form a sub-graph not necessarily connected and is called the co-tree.

Tree compliment: Totality of links i.e. Co-tree.

Independent loop: The addition of each link to a tree, one at a time, results one closed path called an independent loop. Such a loop contains only one link and other tree branches. Obviously, the number of such independent loops equals the number of links.

Tie set: A set of branches contained in a loop such that each loop contains one link and the remainder are tree branches.

Tree branch voltages: The branch voltages may be separated in to tree branch voltages and link voltages. The tree branches connect all the nodes. Therefore if the tree branch voltages are forced to be zero, then all the node potentials become coincident and hence all branch voltages are forced to be zero. As the act of setting only the tree branch voltages to zero forces all voltages in the network to be zero, it must be possible to express all the link voltages uniquely in terms of tree branch voltages. Thus tree branch form an independent set of equations.

Cut set: A set of elements of the graph that dissociates it into two main portions of a network such that replacing any one element will destroy this property. It is a set of branches that if removed divides a connected graph in to two connected sub-graphs. Each cut set contains one tree branch and the remaining being links.

Fig. 2.1 shows a typical network with its graph, oriented graph, a tree, co-tree and a non-planar graph
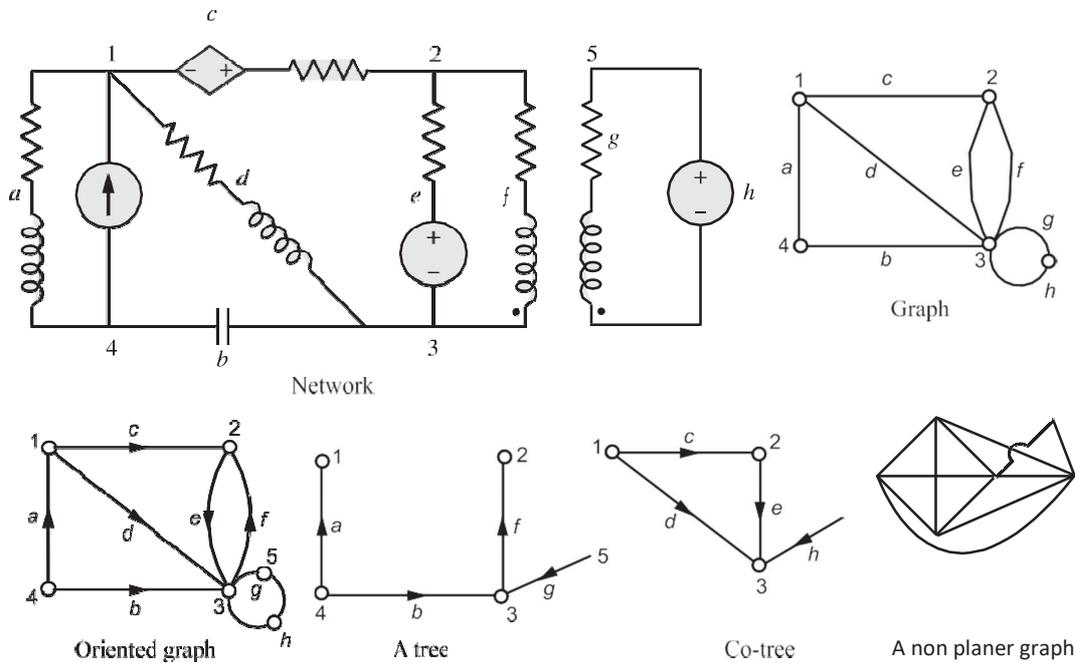
Figure 2.1

Relation between nodes, links, and branches

Let B = Total number of branches in the graph or network

N = total nodes

L = link branches

Then $N-1$ branches are required to construct a tree because the first branch chosen connects two nodes and each additional branch includes one more node.

Therefore number of independent node pair voltages = $N - 1$ = number of tree branches. Then $L = B - (N - 1) = B - N + 1$

Number of independent loops = $B - N + 1$

2.2 Isomorphic graphs

Two graphs are said to be ismorphic if they have the same incidence matrix, though they look dif- ferent. It means that they have the same num- bers of nodes and the same numbers of branches. There is one to one correspondence between the nodes and one to one correspondence between the branches. Fig. 2.2 shows such graphs.
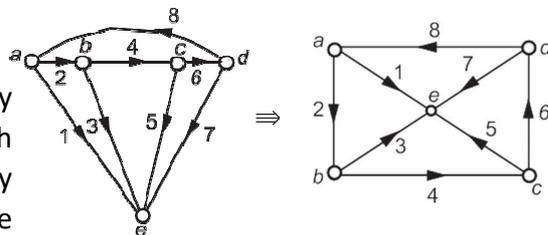
Figure 2.2

For a given oriented graph, there are several representative matrices. They are extremely important in the analytical studies of a graph, particularly in the computer aided analysis and synthesis of large scale networks.

2.2.1 Incidence Matrix A_n

It is also known as augmented incidence matrix. The element node incidence matrix **A** indicates in a connected graph, the incidence of elements to nodes. It is an $N\,B$ matrix with elements of $\mathbf{A}_n = (a_{kj})$

a_{kj} = 1, when the branch b_j is incident to and oriented away from the kth *node.*

= −1, when the branch b_j is incident to and oriented towards the kth node.

= 0, when the branch b_j is not incident to the kth node.

As each branch of the graph is incident to exactly two nodes,

$$\sum_{k=0} a_{kj} = 0 \quad \text{for} \quad j = 1, 2, 3, \cdots B.$$

That is, each column of $\mathbf{A}_n$ has exactly two non zero elements, one being +1 and the other

−1. Sum of elements of any column is zero. The columns of $\mathbf{A}_n$ are lineraly dependent. The rank of the matrix is less than N.

Significance of the incidence matrix lies in the fact that it translates all the geometrical features in the graph into an algebraic expression.

Using the incidence matrix, we can write KCL as

$\mathbf{A}_n\ \mathbf{i}_B$ = 0, where $\mathbf{i}_B$ = branch current vector.

But these equations are not linearly independent. The rank of the matrix **A** is $N\,1$. This property of $\mathbf{A}_n$ is used to define another matrix called reduced incidence matrix or bus incidence matrix.

For the oriented graph shown in Fig. 2.3(a), the incidence matrix is as follows:

Nodes ↓   branches

$$\mathbf{A}_n = \begin{array}{c} a \\ b \\ c \\ d \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ -1 & 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$
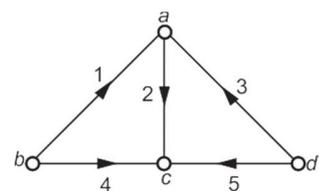
2.2.2 Reduced incidence matrix

Any node of a connected graph can be selected as a reference node. Then the voltages of the other nodes (referred to as buses) can be measured with respect to the assigned reference. The matrix obtained from $\mathbf{A}_n$ by deleting the row corresponding to the reference node is the element- bus incident matrix $\mathbf{A}$ and is called bus incidence matrix with dimension $(N 1) B$. $\mathbf{A}$ is rectangular and therefore singular.

In $\mathbf{A}_n$, the sum of all elements in each column is zero. This leads to an important conclusion that if one row is not known in $\mathbf{A}$, it can be found so that sum of elements of each column must be zero.

From $\mathbf{A}$, we have $\mathbf{A}\, \mathbf{i}_B = 0$, which represents a set of linearly independent equations and there are $N+$ independent node equations.

For the graph shown in Fig 2.3(a), with d selected as the reference node, the reduced incidence matrix is

$$\text{Nodes} \downarrow \quad \begin{matrix} & & \text{branches} & & \\ 1 & 2 & 3 & 4 & 5 \end{matrix}$$

$$\mathbf{A} = \begin{matrix} a \\ b \\ c \end{matrix} \begin{bmatrix} -1 & 1 & -1 & 0 & 0 \\ 0 & -0 & 0 & -1 & -0 \end{bmatrix}$$

Note that the sum of elements of each column in **A** need not be zero
Note that if branch current vector,
Then **A** $\mathbf{i}_B$ = 0 representing a set of independent node equations.

Another important property of **A** is that determinant $\mathbf{AA}^{\mathrm{T}}$ gives the number of possible trees of the network. If **A** = [$\mathbf{A}_t$: $\mathbf{A}_i$] where $\mathbf{A}_t$ and $\mathbf{A}_i$ are sub-matrices of **A** such that $\mathbf{A}_t$ contains only twigs, then det $\mathbf{A}_t$ is either+1 or 1.

To verify the property that det $\mathbf{AA}^{\mathrm{T}}$ gives the number of all possible trees, consider the reduced incidence matrix **A** of the example considered. That is,

$$
\mathbf{A} = \begin{array}{cc}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\left[\begin{array}{ccc:cc}
-1 & 1 & -1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
0 & -1 & 0 & -1 & -1
\end{array} \right] \\
\underbrace{\hspace{3em}}_{\mathbf{A}_t} \quad \underbrace{\hspace{2em}}_{\mathbf{A}_i}
\end{array}
$$

2.3 Cut-set matrix and node pair potentials
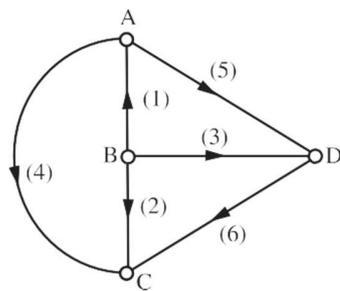
Figure 2.5(a) A directed graph
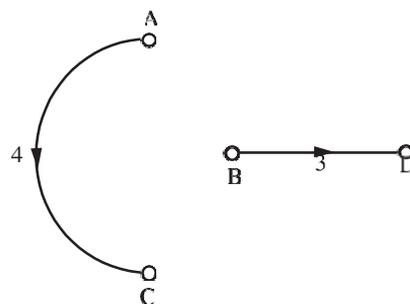
Figure 2.5(b) Two separate graphs created by the cut set {1, 2, 5, 6}

A cut-set of a graph is a set of branches whose removal, cuts the connected graph into two parts such that the replacement of any one branch of the cut-set renders the two parts connected. For example, two separated graphs are obtained for the graph of Fig. 2.5(a) by selecting the cut-set consisting of branches [1, 2, 5, 6]. These seperated graphs are as shown in Fig. 2.5(b).

Just as a systematic method exists for the selection of a set of independent loop current vari- ables, a similar process exists for the selection of a set of independent node pair potential variables.

It is already known that the cut set is a minimal set of branches of the graph, removal of which divides the graph in to two connected sub-graphs. Then it separates the nodes of the graph in to two groups, each being one of the two sub-graphs. Each branch of the tie-set has one of its terminals incident at a node on one sub-graph. Selecting the orientation of the cut set same as that of the tree branch of the cut set, the cut set matrix is constructed row-wise taking one cut set at a time. Without link currents, the network is inactive. In the same way, without node pair voltages the network is active. This is because when one twig voltage is made active with all other twig voltages are zero, there is a set of branches which becomes active. This set is called cut-set. This set is obtained by cutting the graph by a line which cuts one twig and some links. The algebraic sum of these branch currents is zero. Making one twig voltage active in turn, we get entire set of node equations.

This matrix has current values,

$= 1$, if branch is in the cut-set with orientation same as that of tree branch.

$= 1$, if branch is in the cut-set with orientation opposite to that of tree branch.

$= 0$, if branch is not in the cut-set. and

dimension is (1) .

Row-by-row reading, it gives the   at each node and therefore we have **QJ** = 0. The procedure to write cut-set matrix is as follows:

(i) Draw the oriented graph of a network and choose a tree.

(ii) Each tree branch forms an independent cut-set. The direction of this cut-set is same as that of the tree branch. Choose each tree branch in turn to obtain the cut set matrix. Isolate the tree element pairs and energize each bridging tree branch. Assuming the bridging tree branch potential equals the node pair potential, thus regarding it as an independent variable.

(iii) Use the columns of the cut-set matrix to yield a set of equations relating the branch potentials in terms of the node pair potentials. This may be obtained in matrix form as $\mathbf{V} = \mathbf{Q}^T\mathbf{E}$

where and are used to indicate branch potential and node voltage respectively.

In the example shown in Fig 2.5 (c), (3, 4, 5) are tree branches. Links are shown in dotted lines. If two tree branch voltages in 3 and 4 are made zero, the nodes and are at the same potential. Similarly the nodes and are at the same potential. The graph is reduced to the form shown in Fig. 2.5(d) containing only the cut-set branches.

Matrices Associated with Network Graphs

Following are the three matrices that are used in Graph theory.

- Incidence Matrix
- Fundamental Loop Matrix
- Fundamental Cut set Matrix

Incidence Matrix

An Incidence Matrix represents the graph of a given electric circuit or network. Hence, it is possible to draw the graph of that same electric circuit or network from the **incidence matrix**.

We know that graph consists of a set of nodes and those are connected by some branches. So, the connecting of branches to a node is called as incidence. Incidence matrix is represented with the letter A. It is also called as node to branch incidence matrix or **node incidence matrix**.

If there are 'n' nodes and 'b' branches are present in a **directed graph**, then the incidence matrix will have 'n' rows and 'b' columns. Here, rows and columns are corresponding to the nodes and branches of a directed graph. Hence, the **order** of incidence matrix will be **n × b**.

The **elements of incidence matrix** will be having one of these three values, +1, -1 and 0.

- If the branch current is leaving from a selected node, then the value of the element will be +1.
- If the branch current is entering towards a selected node, then the value of the element will be -1.
- If the branch current neither enters at a selected node nor leaves from a selected node, then the value of element will be 0.

Procedure to find Incidence Matrix

Follow these steps in order to find the incidence matrix of directed graph.

- Select a node at a time of the given directed graph and fill the values of the elements of incidence matrix corresponding to that node in a row.
- Repeat the above step for all the nodes of the given directed graph.

Example

Consider the following **directed graph**.
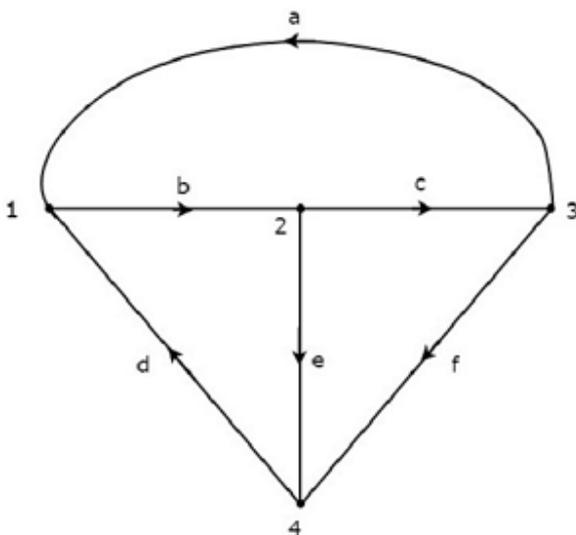
The **incidence matrix** corresponding to the above directed graph will be

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- If the branch current neither enters at a selected node nor leaves from a selected node, then the value of element will be 0.

Procedure to find Incidence Matrix

Follow these steps in order to find the incidence matrix of directed graph.

- Select a node at a time of the given directed graph and fill the values of the elements of incidence matrix corresponding to that node in a row.

- Repeat the above step for all the nodes of the given directed graph.

Fundamental Loop Matrix

Fundamental loop or **f-loop** is a loop, which contains only one link and one or more twigs. So, the number of f-loops will be equal to the number of links. Fundamental loop matrix is represented with letter B. It is also called as **fundamental circuit matrix** and Tie-set matrix. This matrix gives the relation between branch currents and link currents.

If there are 'n' nodes and 'b' branches are present in a **directed graph**, then the number of links present in a co-tree, which is corresponding to the selected tree of given graph will be b-n+1.

So, the fundamental loop matrix will have 'b-n+1' rows and 'b' columns. Here, rows and columns are corresponding to the links of co-tree and branches of given graph. Hence, the order of fundamental loop matrix will be **(b - n + 1) × b**.

The **elements of fundamental loop matrix** will be having one of these three values, +1, -1 and 0.

- The value of element will be +1 for the link of selected f-loop.

- The value of elements will be 0 for the remaining links and twigs, which are not part of the selected f-loop.

- If the direction of twig current of selected f-loop is same as that of f-loop link current, then the value of element will be +1.

- If the direction of twig current of selected f-loop is opposite to that of f-loop link current, then the value of element will be -1.

Procedure to find Fundamental Loop Matrix

Follow these steps in order to find the fundamental loop matrix of given directed graph.

- Select a tree of given directed graph.

- By including one link at a time, we will get one f-loop. Fill the values of elements corresponding to this f-loop in a row of fundamental loop matrix.

- Repeat the above step for all links.

Example

Take a look at the following Tree of **directed graph**, which is considered for incidence matrix.

The above Tree contains three branches d, e & f. Hence, the branches a, b & c will be the links of the Co-Tree corresponding to the above Tree. By including one link at a time to the above Tree, we will get one **f-loop**. So, there will be three **f-loops**, since there are three links. These three f-loops are shown in the following figure.

f-loop 1   f-loop 2   f-loop 3

In the above figure, the branches, which are represented with colored lines form f-loops. We will get the row wise element values of Tie-set matrix from each f-loop. So, the **Tieset matrix** of the above considered Tree will be

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

The rows and columns of the above matrix represents the links and branches of given directed graph. The order of this incidence matrix is 3 × 6.

The **number of Fundamental loop matrices** of a directed graph will be equal to the number of Trees of that directed graph. Because, every Tree will be having one Fundamental loop matrix.

Fundamental Cut-set Matrix

Fundamental cut set or **f-cut set** is the minimum number of branches that are removed from a graph in such a way that the original graph will become two isolated subgraphs. The f-cut set contains only **one twig** and one or more links. So, the number of f-cut sets will be equal to the number of twigs.

Fundamental cut set matrix is represented with letter C. This matrix gives the relation between branch voltages and twig voltages.

If there are 'n' nodes and 'b' branches are present in a **directed graph**, then the number of twigs present in a selected Tree of given graph will be n-1. So, the fundamental cut set matrix will have 'n-1' rows and 'b' columns. Here, rows and columns are corresponding to the twigs of selected tree and branches of given graph. Hence, the **order** of fundamental cut set matrix will be **(n-1) × b**.

The **elements of fundamental cut set matrix** will be having one of these three values, +1, -1 and 0.

- The value of element will be +1 for the twig of selected f-cutset.
- The value of elements will be 0 for the remaining twigs and links, which are not part of the selected f-cutset.
- If the direction of link current of selected f-cut set is same as that of f-cutset twig current, then the value of element will be +1.
- If the direction of link current of selected f-cut set is opposite to that of f-cutset twig current, then the value of element will be -1.

Procedure to find Fundamental Cut-set Matrix

Follow these steps in order to find the fundamental cut set matrix of given directed graph.

- Select a Tree of given directed graph and represent the links with the dotted lines.
- By removing one twig and necessary links at a time, we will get one f-cut set. Fill the values of elements corresponding to this f-cut set in a row of fundamental cut set matrix.
- Repeat the above step for all twigs.

Fundamental Loop Matrix

Fundamental loop or **f-loop** is a loop, which contains only one link and one or more twigs. So, the number of f-loops will be equal to the number of links. Fundamental loop matrix is represented with letter B. It is also called as **fundamental circuit matrix** and Tie-set matrix. This matrix gives the relation between branch currents and link currents.

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- The value of element will be +1 for the link of selected f-loop.
- The value of elements will be 0 for the remaining links and twigs, which are not part of the selected f-loop.
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Procedure to find Fundamental Loop Matrix

Follow these steps in order to find the fundamental loop matrix of given directed graph.

- Select a tree of given directed graph.
- By including one link at a time, we will get one f-loop. Fill the values of elements corresponding to this f-loop in a row of fundamental loop matrix.
- Repeat the above step for all links.

Example

Take a look at the following Tree of **directed graph**, which is considered for incidence matrix.

The above Tree contains three branches d, e & f. Hence, the branches a, b & c will be the links of the Co-Tree corresponding to the above Tree. By including one link at a time to the above Tree, we will get one **f-loop**. So, there will be three **f-loops**, since there are three links. These three f-loops are shown in the following figure.

f-loop 1   f-loop 2   f-loop 3

In the above figure, the branches, which are represented with colored lines form f-loops. We will get the row wise element values of Tie-set matrix from each f-loop. So, the **Tieset matrix** of the above considered Tree will be

$$B=\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 1 \end{bmatrix} B=[100-10-10101100010-11]$$

The rows and columns of the above matrix represents the links and branches of given directed graph. The order of this incidence matrix is 3 × 6.

The **number of Fundamental loop matrices** of a directed graph will be equal to the number of Trees of that directed graph. Because, every Tree will be having one Fundamental loop matrix.

Fundamental Cut-set Matrix

Fundamental cut set or **f-cut set** is the minimum number of branches that are removed from a graph in such a way that the original graph will become two isolated subgraphs. The f-cut set contains only **one twig** and one or more links. So, the number of f-cut sets will be equal to the number of twigs.

Fundamental cut set matrix is represented with letter C. This matrix gives the relation between branch voltages and twig voltages.

If there are 'n' nodes and 'b' branches are present in a **directed graph**, then the number of twigs present in a selected Tree of given graph will be n-1. So, the fundamental cut set matrix will have 'n-1' rows and 'b' columns. Here, rows and columns are corresponding to the twigs of selected tree and branches of given graph. Hence, the **order** of fundamental cut set matrix will be **(n-1) × b**.

The **elements of fundamental cut set matrix** will be having one of these three values, +1, -1 and 0.

- The value of element will be +1 for the twig of selected f-cutset.
- The value of elements will be 0 for the remaining twigs and links, which are not part of the selected f-cutset.
- If the direction of link current of selected f-cut set is same as that of f-cutset twig current, then the value of element will be +1.
- If the direction of link current of selected f-cut set is opposite to that of f-cutset twig current, then the value of element will be -1.

Procedure to find Fundamental Cut-set Matrix

Follow these steps in order to find the fundamental cut set matrix of given directed graph.

- Select a Tree of given directed graph and represent the links with the dotted lines.
- By removing one twig and necessary links at a time, we will get one f-cut set. Fill the values of elements corresponding to this f-cut set in a row of fundamental cut set matrix.
- Repeat the above step for all twigs.

Example

Consider the same **directed graph** , which we discussed in the section of incidence matrix. Select the branches d, e & f of this directed graph as twigs. So, the remaining branches a, b & c of this directed graph will be the links.

The **twigs** d, e & f are represented with solid lines and **links** a, b & c are represented with dotted lines in the following figure.
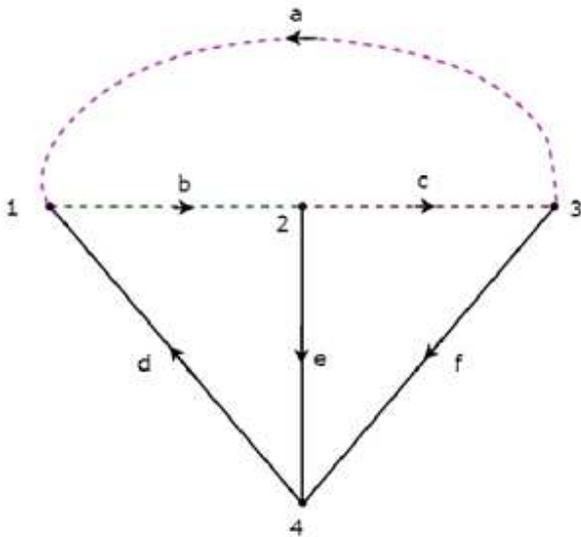
By removing one twig and necessary links at a time, we will get one f-cut set. So, there will be three f-cut sets, since there are three twigs. These three **f-cut sets** are
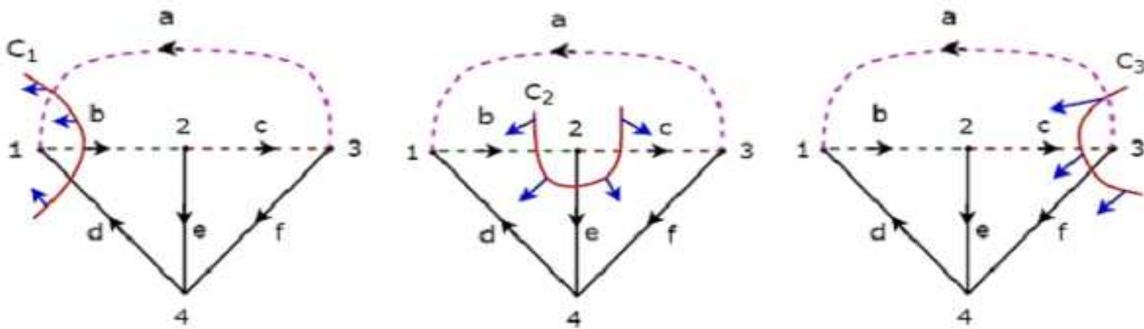
shown in the following figure.

We will be having three f-cut sets by removing a set of twig and links of C_1, C_2 and C_3. We will get the row wise element values of fundamental cut set matrix from each f-cut set.

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MODULE-III
MAGNETIC CIRCUITS

Magnetic Circuits:

Introduction to the Magnetic Field:

Magnetic fields are the fundamental medium through which energy is converted from one form to another in motors, generators and transformers. Four basic principles describe how magnetic fields are used in these devices.

0. A current-carrying conductor produces a magnetic field in the area around it.
    Explained in Detail by Fleming's Right hand rule and Amperes Law.
1. A time varying magnetic flux induces a voltage in a coil of wire if it passes through that coil.

    (basis of Transformer action)

    Explained in detail by the Faradays laws of Electromagnetic Induction.
2. A current carrying conductor in the presence of a magnetic field has a force induced in it (Basis of Motor action)
3. A moving wire in the presence of a magnetic field has a voltage induced in it (Basis of Generator action)

    We will be studying in this unit the first two principles in detail and the other two principles in the next unit on DC machines.

Two basic laws governing the production of a magnetic field by a current carrying conductor :

The direction of the magnetic field produced by a current carrying conductor is given by the **Flemings Right hand rule** and its' amplitude is given by the **Ampere's Law**.

Flemings right hand rule: *Hold the conductor carrying the current in your right hand such that the Thumb points along the wire in the direction of the flow of current, then the fingers will encircle the wire along the lines of the Magnetic force.*
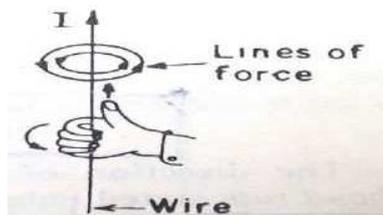
Ampere's Law : *The line integral of the magnetic field intensity H around a closed magnetic path is equal to the total current enclosed by the path*.

This is the basic law which gives the relationship between the Magnetic field Intensity H and the current I and is mathematically expressed as

$$H.\,dl = I_{net}$$

where **H** is the magnetic field intensity produced by the current **I$_{net}$** and *dl* is a differential element of length along the path of integration. **H** is measured in *Ampere-turns per meter*.

Important parameters and their relation in magnetic circuits :

- Consider a current carrying conductor wrapped around a ferromagnetic core as shown in the figure below .
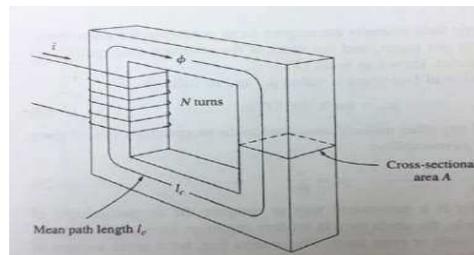
- Applying Ampere's law, the total amount of magnetic field induced will be proportional to the amount of current flowing through the conductor wound with N turns around the ferromagnetic material as shown. Since the core is made of ferromagnetic material, it is assumed that a majority of the magnetic field will be confined to the core.
- The path of integration in this case as per the Ampere's law is the mean path length of the core, **l$_C$**. The current passing within the path of integration **I$_{net}$** is then **Ni**, since the coil of wire cuts the path of integration **N** times while carrying the current **i**. Hence Ampere's Law becomes :
Hl$_c$ = Ni

  Therefore   **H = Ni/l$_c$**

- In this sense, **H** (Ampere turns per meter) is known as the effort required to induce a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. Thus: **B = μH** where

  B = magnetic flux density [webers per square meter, or Tesla (T)]

  μ= magnetic permeability of material (Henrys per meter)

  H = magnetic field intensity (ampere-turns per meter)

- The constant **μ** may be further expanded to include *relative permeability* which can be defined as below:
μ$_r$ =   μ /μ$_o$

where μ_o = permeability of free space (equal to that of air)

- Hence the permeability value is a combination of the relative permeability and the permeability of free space. The value of relative permeability is dependent upon the type of material used. The higher the amount permeability, the higher the amount of flux induced in the core. Relative permeability is a convenient way to compare the magnetizability of materials.
- Also, because the permeability of iron is so much higher than that of air, the majority of the flux in an iron core remains inside the core instead of travelling through the surrounding air, which has lower permeability. The small leakage flux that does leave the iron core is important in determining the flux linkages between coils and the self-inductances of coils in transformers and motors.
- In a core such as shown in the figure above

$$B = \mu H = \mu \ Ni/l_c$$

Now, to measure the total flux flowing in the ferromagnetic core, consideration has to

be made in terms of its cross sectional area (CSA). Therefore:

$\Phi = B. \ dA$ where: **A** = cross sectional area throughout the core.

Assuming that the flux density in the ferromagnetic core is constant throughout hence

the equation simplifies to: $\Phi = $ **B.A**

Taking the previous expression for **B**   we get $\Phi = \mu \ NiA/l_c$

Electrical analogy of magnetic circuits:

The flow of magnetic flux induced in the ferromagnetic core is analogous to the flow of electric current in an electrical circuit hence the name magnetic circuit.

The analogy is as follows:
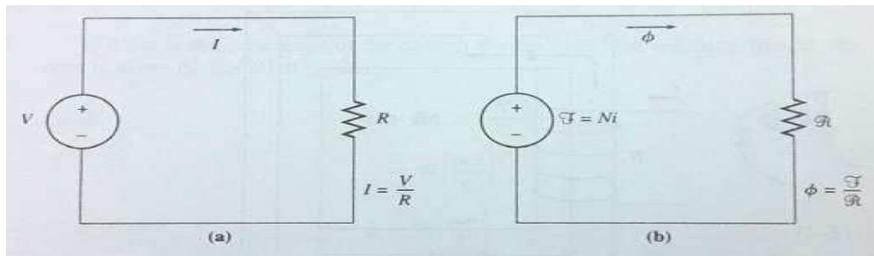
(a) Electric Circuit   **(b) Electrical Analogy of Magnetic Circuit**

Referring to the magnetic circuit analogy, **F** is denoted as **magnetomotive force** (mmf) which is similar to Electromotive force in an electrical circuit (emf). Therefore, we can say that **F** is the force which pushes magnetic flux around a ferromagnetic core with a value of **Ni** (refer to ampere's law). Hence **F** is measured in ampere turns. Hence the magnetic circuit equivalent equation is as shown:

$$F = \emptyset.R \text{ (similar to V=IR)}$$

We already have the relation $\Phi = \mu \, NiA/l$ and using this we get $R = F / \Phi = Ni/ \Phi$

$$R = Ni /(\mu \, NiA/l) = l/ \mu \, A$$

- The polarity of the mmf will determine the direction of flux. To easily determine the direction of flux, the *'right hand curl'* rule is applied:
  When the direction of the curled fingers indicates the direction of current flow the resulting thumb direction will show the magnetic flux flow.
- The element of **R** in the magnetic circuit analogy is similar in concept to the electrical resistance. It is basically the measure of material resistance to the flow of magnetic flux. **Reluctance** in this analogy obeys the rule of electrical resistance (Series and Parallel Rules). Reluctance is measured in ***Ampere-turns per weber***.
- The inverse of electrical resistance is conductance which is a measure of conductivity of a material. Similarly the inverse of reluctance is known as **permeance P** which represents the degree to which the material permits the flow of magnetic flux.

- By using the magnetic circuit approach, calculations related to the magnetic field in a ferromagnetic material are simplified but with a little inaccuracy.

Equivalent Reluctance of a series Magnetic circuit : $R_{eqseries} = R_1 + R_2 + R_3 + ...$

Equivalent Reluctance of a Parallel Magnetic circuit: $1/R_{eqparallel} = 1/R_1 + 1/R_2 + 1/R_3 + ...$

Electromagnetic Induction and Faraday's law – Induced Voltage from a Time-Changing

Magnetic Field:

Faraday's Law:

Whenever a varying magnetic flux passes through a turn of a coil of wire, voltage will be induced in the turn of the wire that is directly proportional to the rate of change of the flux linkage with the turn of the coil of wire.

$$e_{ind} \propto -d\emptyset/dt$$
$$e_{ind} = -k. \, d\emptyset/dt$$

The negative sign in the equation above is in accordance to **Lenz' Law** which states:

The direction of the induced voltage in the turn of the coil is such that if the coil is short circuited, it would produce a current that would cause a flux which opposes the original change of flux.

And **k** is the constant of proportionality whose value depends on the system of units chosen. In the SI system of units **k=1** and the above equation becomes:

$$e_{ind} = - \, d\emptyset/dt$$

Normally a coil is used with several turns and if there are N number of turns in the coil with the same amount of flux flowing through it then: $e_{ind} = - \, N \, d\emptyset/dt$

*Change in the flux linkage **NØ** of a coil can be obtained in two ways:*

1. Coil remains stationary and flux changes with time (Due to AC current like in Transformers and this is called Statically induced e.m.f)
2. Magnetic flux remains constant and stationary in space, but the coil moves relative to the magnetic field so as to create a change in the flux linkage of the coil (Like in Rotating machines and this is a called Dynamically induced e.m.f.

Self inductance:

From the Faradays laws of Electromagnetic Induction we have seen that an e.m.f will be induced in a conductor when a time varying flux is linked with a conductor and the amplitude of the induced e.m.f is proportional to the rate of change of the varying flux.

If the time varying flux is produced by a coil of **N** turns then the coil itself links with the time varying flux produced by itself and an emf will be induced in the same coil. This is called self inductance .

The flux **Ø** produced by a coil of N turns links with its own N turns of the coil and hence the total flux linkage is equal to **NØ = (μ N² A / l) I** *[using the expression Φ = μ NiA/l we already*

*develop*ed] Thus we see that the total magnetic flux produced by a coil of N turns and linked with itself is proportional to the current flowing through the coil i.e.

$$N\emptyset \propto \quad \text{or} \quad N\emptyset = L\,I$$

From the Faradays law of electromagnetic Induction, the self induced e.m.f for this coil of N turns is given by:

$$e_{ind} = -N\,d\emptyset/dt = -L\,dI/dt$$

The constant of proportionality **L** is called the self Inductance of the coil or simply Inductance and its value is given by **L = (μ N² A / l)**. If the radius of the coil is **r** then:

$$L = (\mu\,N^2\,\pi r^2\,/\,l)\,i$$

From the above two equations we can see that Self Inductance of a coil can be defined as the flux produced per unit current i.e *Weber/Ampere* (equation1) or the induced emf per unit rate of change of current i.e *Volt-sec/Ampere* (equation 2)

The unit of Inductance is named after Joseph Henry as **Henry** and is given to these two combinations as :

$$1H = 1WbA^{-1} = 1VsA^{-1}$$

Self Inductance of a coil is defined as one Henry if an induced emf of one volt is generated when the current in the coil changes at the rate of one Ampere per second.

Henry is relatively a very big unit of Inductance and we normally use Inductors of the size of mH (10^{-3} H) or μH (10^{-3}H)

Mutual inductance and Coefficient of coupling:

In the case of Self Inductance an emf is induced in the same coil which produces the varying magnetic field. The same phenomenon of Induction will be extended to a separate second coil if it is located in the vicinity of the varying magnetic field produced by the first coil. Faradays law of electromagnetic Induction is equally applicable to the second coil also. A current flowing in one coil establishes a magnetic flux about that coil and also about a second coil nearby but of course with a lesser intensity. The time-varying flux produced by the first coil and surrounding the second coil produces a voltage across the terminals of the second coil. This voltage is proportional to the time rate of change of the current flowing through the first coil.

Figure (*a)* shows a simple model of two coils L_1 and L_2, sufficiently close together that the flux produced by a current $i_1(t)$ flowing through L_1 establishes an open-circuit voltage $v_2(t)$ across the terminals of L_2.***Mutual inductance,M*21**, is defined such that

$$v_2(t) = M_{21}di_1(t)/dt \text{ --------------------[1]}$$
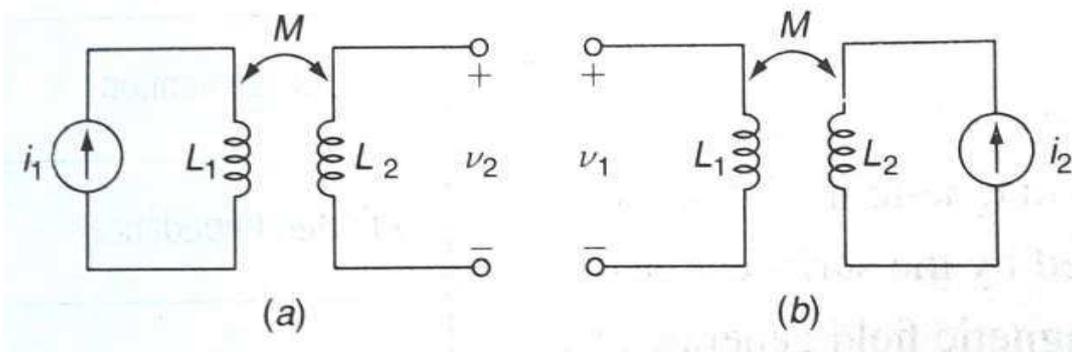
Figure 4.17 (*a*) A current *i*1 through *L*1 produces an open-circuit voltage *v*2across *L*2. (*b*) A current *i*2 through *L*2 produces an open-circuit voltage *v*1 across *L*1.

The order of the subscripts on M_{21} indicates that a voltage response is produced at L_2 by a current source at L_1. If the system is reversed, as indicated

in fig.(*b*) then we have

$$v_1(t) = M_{12}di_2(t)/dt \text{------------------- [2]}$$

It can be proved that the two mutual inductances M_{12} and M_{21} are equal and thus, $M_{12} = M_{21} = M$. The existence of mutual coupling between two coils is indicated by a double-headed arrow, as shown in Fig. (*a*)and (*b*)

Mutual inductance is measured in Henrys and, like resistance, inductance, and capacitance, is a positive quantity. The voltage *M di/dt*, however, may appear as either a positive or a negative quantity depending on whether the current is increasing or decreasing at a particular instant of time.

 Coefficient of coupling k **:** Is given by the relation **M = k√L₁ L₂** and its value lies between 0 and 1. It can assume the maximum value of 1 when the two coils are wound on the same core such that flux produced by one coil completely links with the other coil. This is possible in well designed cores with high permeability. Transformers are designed to achieve a coefficient of coupling of 1.

Dot Convention:

The polarity of the voltage induced in a coil depends on the sense of winding of the coil. In the case of Mutual inductance it is indicated by use of a method called "***dot convention".*** The dot

convention makes use of a large dot placed at one end of each of the two coils which are mutually coupled. Sign of the mutual voltage is determined as follows:

A current entering the dotted terminal of one coil produces an open circuit voltage with a positive voltage reference at the dotted terminal of the second coil.

Thus in Fig(a) i_1 enters the dotted terminal of L_1, v_2 is sensed positively at the dotted terminal of L_2, and $v_2 = M \, di_1/dt$.

It may not be always possible to select voltages or currents throughout a circuit so that the passive sign convention is everywhere satisfied; the same situation arises with mutual coupling. For example, it may be more convenient to represent v_2 by a positive voltage reference at the undotted terminal, as shown in Fig (b). Then $v_2 = -M \, di_1/dt$. Currents also may not always enter the dotted terminal as indicated by Fig (c) and (d). Then we note that:

A current entering the undotted terminal of one coil provides a voltage that is positively sensed at the undotted terminal of the second coil.
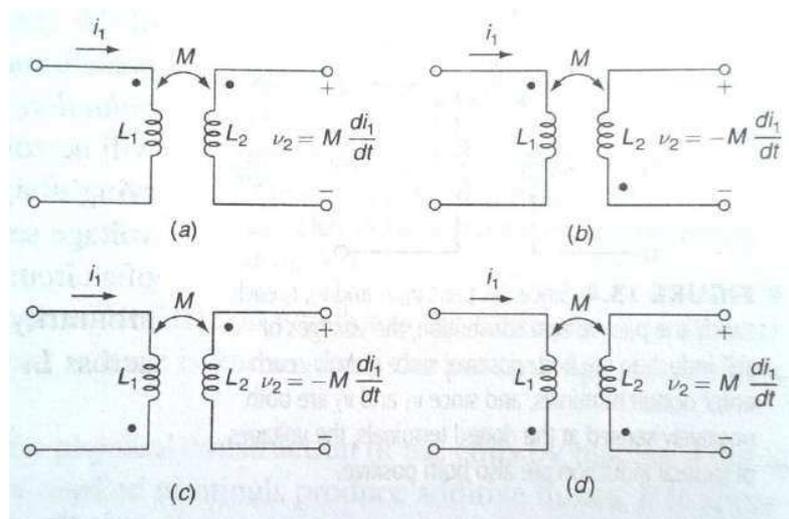
Figure 4.18 : (a) and (b) Current entering the dotted terminal of one coil produces a voltage that is sensed positively at the dotted terminal of the second coil. **(c) and (d)** Current entering the undotted terminal of one coil produces a voltage that is sensed positively at the undotted terminal of the second coil.

Important Concepts and formulae:

Resonance and Series RLC circuit:

$\omega_r{}^2 = \omega_1\omega_2 = 1/LC \therefore \omega_r = \sqrt{\omega_1\omega_2} = 1/\sqrt{LC}$

$BW = R/2\pi L$

$Q = \omega_r L / R = 1/ \omega_r RC$ **and in terms of R,L and C** $= (1/R) (\sqrt{L/C})$

$Q = f_r / BW$ **i.e.** inversely proportional to the BW

Voltage magnification Magnification $= Q = V_L/V$ **or** V_C / V

Important points In Series RLC circuit at resonant frequency:

- The impedance of the circuit becomes purely resistive and minimum i.e **Z = R**
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is **Q** times higher than the voltage across the resistor

Resonance and Parallel RLC circuit:

$\omega_r^2 = \omega_1\omega_2 = 1/LC$ $\therefore$ $\omega_r = \sqrt{\omega_1\omega_2} = 1/\sqrt{LC}$ same as in series RLC circuit

BW = 1/2π RC

$Q = R /\omega_r L = \omega_r RC$ **and in terms of R, L and C** $= R (\sqrt{C/L})$ [Inverse of what we got in Series RLC circuit]

$Q = f_r / BW$ In Parallel RLC also inversely proportional to the BW

Current Magnification $= Q = I_L/I$ **or** I_C / I

Important points In Parallel RLC circuit at resonant frequency :

- The impedance of the circuit becomes resistive and maximum i.e **Z = R**
- The current in the circuit becomes minimum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The current through the Capacitor becomes equal and opposite to the current through the Inductor at resonance and is Q times higher than the current through the resistor

Magnetic circuits :

Ampere's Law: $H. dl = I_{net}$ and in the case of a simple closed magnetic path of a ferromagnetic material it simplifies to *Hl= Ni* *or H = Ni/l*

Magnetic flux density: **B = μH**

Magnetic field intensity: **H = Ni/l**

Total magnetic flux intensity**:** **Ø = BA = μHA = μ Ni A / l**

Reluctance of the magnetic circuit: $R = mmf/Flux = Ni/ \emptyset = l/\mu A$

Faradays law of electromagnetic Induction:

Self induced e.m.f of a coil of N turns is given by: $e_{ind} = - N \, d\emptyset/dt = -L \, dI/dt$ where L is the inductance of the coil of N turns with radius r and given by $L = (\mu N^2 \pi r^2 / l) \, i$

Equivalent Reluctance of a series Magnetic circuit: $R_{eqseries} = R_1 + R_2 + R_3 + ...$

Equivalent Reluctance of a Parallel Magnetic circuit: $1/R_{eqparallel} = 1/R_1 + 1/R_2 + 1/R_3 + ..$

Coefficient of coupling k Is given by the relation: $M = k\sqrt{L_1 \, L_2}$

Illustrative examples:

Example 1: A toroidal core of radius 6 cms is having 1000 turns on it. The radius of cross section of the core 1cm.Find the current required to establish a total magnetic flux of 0.4mWb.When

(a) The core is nonmagnetic
(b) The core is made of iron having a relative permeability of 4000
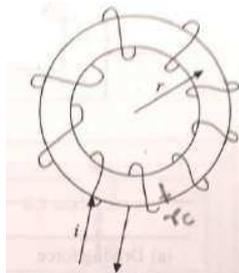
Solution:

This problem can be solved by the direct application of the following formulae we know in magnetic circuits: $B = \Phi/A = \mu H$ and $H = Ni/l$

Where

B = magnetic flux density (Wb/mtr^2) Φ = Total magnetic flux (Wb))

A = Cross sectional area of the core(mtr^2) μ = $\mu_r\mu_0$ = Permeability (Henrys/mtr) μ_r = Relative permeability of the material (Dimensionless)

μ_0 = free space permeability = $4\pi \times 10^{-7}$ Henrys/mtr

H = Magenetic field intensity AT/mtr   N = Number of turns of the coil

i   = Current in the coil (Amps)   l = Length of the coil (mtrs)

from the above relations we can get **i** as

$$i = H\, l/N = (1/\mu)(\Phi/A)\, l/N = (1/\mu)(\Phi/N)\, l/A = (1/\mu)(\Phi/N) * 2\pi r_T / \pi r^2] = [2r\, \Phi / \mu N r_C^2]$$

Where r_T is the radius of the toroid and r_C is the radius of cross section of the coil

Now we can calculate the currents in the two cases by substituting the respective values.

(a) Here $\mu = \mu_0$.   Therefore i = (2 x 6 x 10^{-2} x 4 x 10^{-4})/ (4π x 10^{-7} x 1000 x 10^{-4}) = 380 Amps
(b) Here $\mu = \mu_r\mu_0$. Therefore i = (2 x 6 x 10^{-2} x 4 x 10^{-4})/ (4000 x4π x 10^{-7} x 1000 x 10^{-4}) = 0.095 Amps

Ex.2: (a) Draw the electrical equivalent circuit of the magnetic circuit shown in the figure below. The area of the core is 2x2 cm^2 .The length of the air gap is 1cm and lengths of the other limbs are shown in the figure. The relative permeability of the core is 4000.

(b) Find the value of the current '**i**' in the above example which produces a flux density of 1.2 Tesla in the air gap . The number f turns of the coil are 5000.
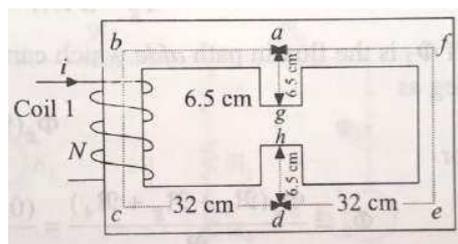
Solution: (a)

To draw the equivalent circuit we have to find the Reluctances of the various flux paths independently.

The reluctance of the path *abcd* is given by: R_1 = length of the path *abcd* /$\mu_r\mu_0 A$

= (32x10^{-2}) / (4π x 10^{-7} x 4000 x 4 x 10^{-4})   =1.59 x 10^5 AT/Wb

The reluctance of the path *afed* is equal to the reluctance of the path *abcd* since it has the same length, same permeability and same cross sectional area. Thus $R_1 = R_2$

Similarly the reluctance of the path *ag* (R_3) is equal to that of the path *hd* (R_4) and can be calculated as: $R_3 = R_4 = (6.5 \times 10^{-2}) / (4\pi \times 10^{-7} \times 4000 \times 4 \times 10^{-4})$ $= 0.32 \times 10^5$ AT/Wb

The reluctance of the air gap path *gh* R_G can be calculated as : R_G = length of the air gap path $gh/\mu_0 A$

(Here it is to be noted that μ is to be taken as μ_0 only and μ_r should not be included)

$R_G = (1 \times 10^{-2}) / (4\pi \times 10^{-7} \times 4 \times 10^{-4}) = 198.94 \times 10^5$ AT/Wb

The equivalent electrical circuit is shown in the figure below with the values of the reluctances as given below the circuit diagram.
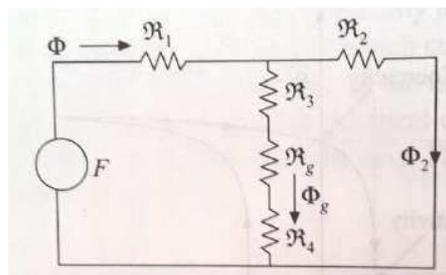
$R_1 = R_2 = 1.59 \times 10^5$ AT/Wb $R_3 = R_4 = 0.32 \times 10^5$ AT/Wb $R_G = 198.94 \times 10^5$ AT/Wb

Solution: (b)   This problem is solved in the following steps:

1. **First the flux through the air gap Φ_G is found out.** The flux in the air gap Φ_G is given by the product of the Flux density in the air gap **B** and the cross sectional area of the core in that region **A** . Hence Φ_G = **B.A** = $1.2 \times 4 \times 10^{-4}$ = **0.00048 Wb**
   It is to be noted here that the same flux would be passing through the reluctances **R_3, R_G & R_4**
2. **Next, the Flux in the path *afed* Φ_2 is to be found out .** This can be found out by noticing that the mmf across the reluctance R_2 is same as the mmf across the sum of the reluctances **R_3, R_G, and R_4** coming in parallel with **R_4**. Hence by equating them we get

   $\Phi_G (R_3 + R_G + R_4) = \Phi_2 R_2$ from which we get $\Phi_2 = \Phi_G (R_3 + R_G + R_4) / R_2$

   Hence $\Phi_2 = [0.00048 \times (0.32 + 198.94 + 0.32) \times 10^5] / 1.59 \times 10^5$ = **0.06025 Wb**

3. **Next , the total flux Φ flowing through the reluctance of the path abcd R_1 produced by the winding is to be found out**. This is the sum of the air gap flux $Φ_G$ and the flux in the outer limb of the core $Φ_2$: i.e $Φ = Φ_G + Φ_2 = ($0.00048 + 0.06025$) = $**0.0607 Wb**

4. **Next , The total mmf F given by F = Ni is to be found out** . This is also equal to the sum of the mmfs across the reluctances R_1 and R_2 **[or ($R_3 + R_G + R_4$)]** = **Φ R_1 + $Φ_2$ R_2** from which we can get **'i'** as : **'i'** = (**Φ R_1 + $Φ_2$ R_2**) **/ N** = [0.0607 x 1.59×10^5 + 0.06025 x 1.59×10^5]/5000 = 3.847 Amps

is = 3.847 Amps

MODULE-IV
SINGLE PHASE AC CIRCUITS

Introduction to Single Phase AC Circuit:

In a dc circuit the relationship between the applied voltage V and current flowing through the circuit I is a simple one and is given by the expression I = V/R but in an a c circuit this simple relationship does not hold good. Variations in current and applied voltage set up magnetic and electrostatic effects respectively and these must be taken into account with the resistance of the circuit while determining the quantitative relations between current and applied voltage.

With comparatively low-voltage, heavy- current circuits magnetic effects may be very large, but electrostatic effects are usually negligible. On the other hand with high-voltage circuits electrostatic effects may be of appreciable magnitude, and magnetic effects are also present.

Here it has been discussed how the magnetic effects due to variations in current do and electrostatic effects due to variations in the applied voltage affect the relationship between the applied voltage and current.

---

2. Purely Resistive Circuit:

ADVERTISEMENTS:

A purely resistive or a non-inductive circuit is a circuit which has inductance so small that at normal frequency its reactance is negligible as compared to its resistance. Ordinary filament lamps, water resistances etc., are the examples of non-inductive resistances. If the circuit is purely non-inductive, no reactance emf (i.e., self- induced or back emf) is set up and whole of the applied voltage is utilised in overcoming the ohmic resistance of the circuit.

Consider an ac circuit containing a non-inductive resistance of R ohms connected across a sinusoidal voltage represented by v = V sin wt, as shown in Fig. 4.1 (a).

MREC(A)

(a) Circuit Diagram

(c) Phasor Diagram

(b) Wave Diagram
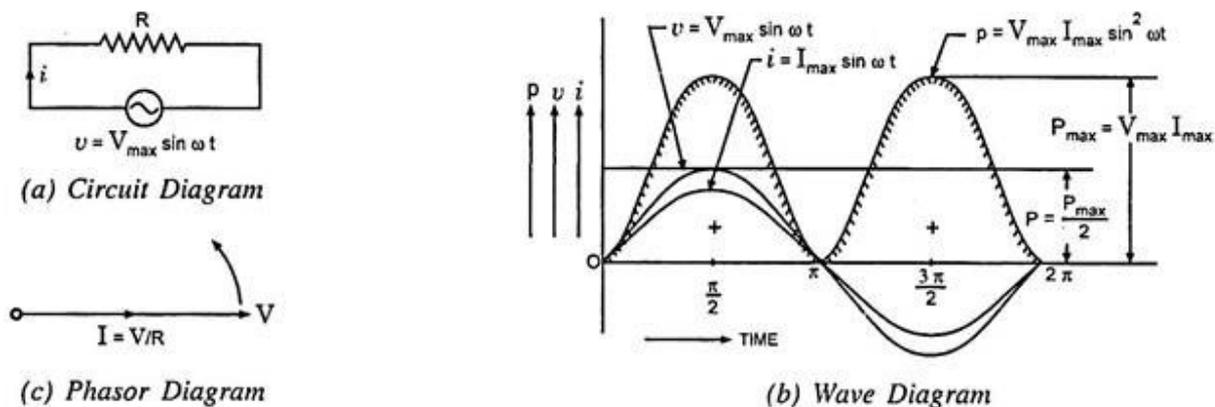
Fig. 4.1 *Purely Resistive Circuit*

As already said, when the current flowing through a pure resistance changes, no back emf is set up, therefore, applied voltage has to overcome the ohmic drop of i R only:

$$i.e. \quad iR = v$$

$$\text{or} \quad i = \frac{v}{R} = \frac{V_{max}}{R} \sin \omega t$$

Current will be maximum when $\omega t = \dfrac{\pi}{2}$ or $\sin \omega t = 1$

$$\therefore \quad I_{max} = \frac{V_{max}}{R}$$

And instantaneous current may be expressed as:

$i = I_{max} \sin \omega t$

From the expressions of instantaneous applied voltage and instantaneous current, it is evident that in a pure resistive circuit, the applied voltage and current are in phase with each other, as shown by wave and phasor diagrams in Figs. 4.1 (b) and (c) respectively.

Power in Purely Resistive Circuit:

ADVERTISEMENTS:

The instantaneous power delivered to the circuit in question is the product of the instantaneous values of applied voltage and current.

MREC(A)

$$i.e. \quad p = v\,i = V_{max}\sin\omega t\,I_{max}\sin\omega t = V_{max}\,I_{max}\sin^2\omega t$$

$$\text{or} \quad p = \frac{V_{max}\,I_{max}}{2}(1-\cos 2\omega t) \qquad\qquad \text{Since } \sin^2\omega t = \frac{1-\cos 2\omega t}{2}$$

$$= \frac{V_{max}\,I_{max}}{2} - \frac{V_{max}\,I_{max}}{2}\cos 2\omega t$$

$$\text{Average power, } P = \text{Average of } \frac{V_{max}\,I_{max}}{2} - average\ of\ \frac{V_{max}\,I_{max}}{2}\cos 2\omega t$$

Since average of $\dfrac{V_{max}\,I_{max}}{2}\cos 2\omega t$ over a complete cycle is zero,

$$P = \frac{V_{max}\,I_{max}}{2} = \frac{V_{max}}{\sqrt{2}}\cdot\frac{I_{max}}{\sqrt{2}} = V\,I \text{ watts}$$

Where V and I are the rms values of applied voltage and current respectively.

Thus for purely resistive circuits, the expression for power is the same as for dc circuits. From the power curve for a purely resistive circuit shown in Fig. 4.1 (b) it is evident that power consumed in a pure resistive circuit is not constant, it is fluctuating.

However, it is always positive. This is so because the instantaneous values of voltage and current are always either positive or negative and, therefore, the product is always positive. This means that the voltage source constantly delivers power to the circuit and the circuit consumes it.

---

3. Purely Inductive Circuit:

An inductive circuit is a coil with or without an iron core having negligible resistance. Practically pure inductance can never be had as the inductive coil has always small resistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance arid is known as a choke coil.

When an alternating voltage is applied to a purely inductive coil, an emf, known as self-induced emf, is induced in the coil which opposes the applied voltage. Since coil has no resistance, at every instant applied voltage has to overcome this self-induced emf only.

Let the applied voltage $v = V_{max} \sin \omega t$

and self inductance of coil = L henry

Self induced emf in the coil, $e_L = -L \dfrac{di}{dt}$

Since applied voltage at every instant is equal and opposite to the self induced emf i.e. $v = -e_L$

$$\therefore \quad V_{max} \sin \omega t = -\left(-L\dfrac{di}{dt}\right)$$

$$\text{or} \quad di = \dfrac{V_{max}}{L} \sin \omega t \, dt$$

Integrating both sides we get

$$i = \dfrac{V_{max}}{L} \int \sin \omega t \, dt = \dfrac{V_{max}}{\omega L} (-\cos \omega t) + A$$

where A is a constant of integration, which is found to be zero from initial conditions

$$\text{i.e.} \quad i = \dfrac{-V_{max}}{\omega L} \cos \omega t = \dfrac{V_{max}}{\omega L} \sin\left(\omega t - \dfrac{\pi}{2}\right)$$

Current will be maximum when $\sin\left(\omega t - \dfrac{\pi}{2}\right) = 1$, hence, maximum value of current,

$$I_{max} = \dfrac{V_{max}}{\omega L}$$

and instantaneous current may be expressed as $i = I_{max} \sin\left(\omega t - \dfrac{\pi}{2}\right)$

From the expressions of instantaneous applied voltage and instantaneous current flowing through a purely inductive coil it is observed that the current lags behind the applied voltage by π/2 as shown in Fig. 4.2 (b) by wave diagram and in Fig 4.2 (c) by phasor diagram.

MREC(A)

(a) Circuit Diagram

(c) Phasor Diagram

(b) Wave Diagram
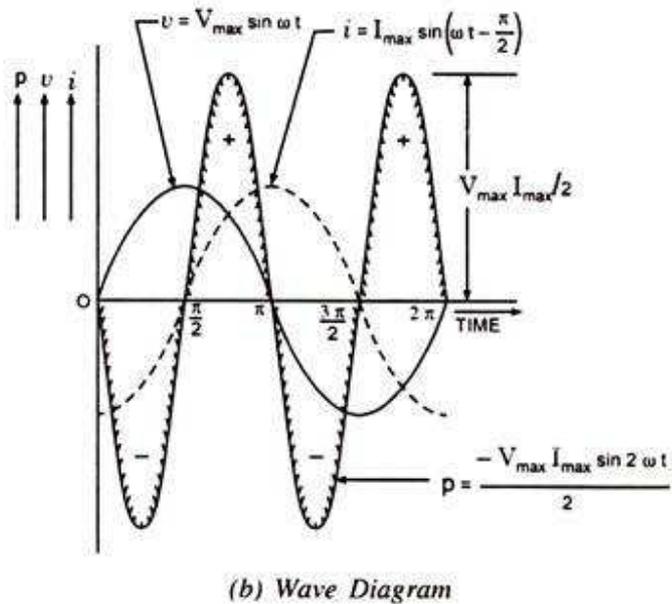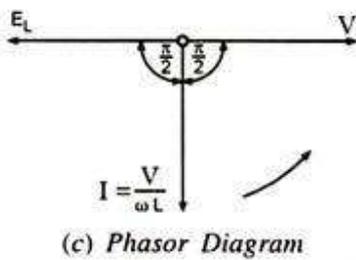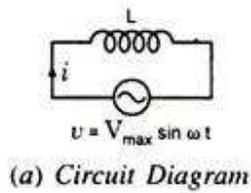
Fig. 4.2 Purely Inductive Circuit

Inductive Reactance:

ωL in the expression $I_{max} = V_{max}/\omega L$ is known as inductive reactance and is denoted by X_L i.e., $X_L = \omega L$
If L is in henry and co is in radians per second then X_L will be in ohms.

Power in Purely Inductive Circuit:

Instantaneous power, $p = v \times i = V_{max} \sin \omega t\, I_{max} \sin(\omega t - \pi/2)$

Or $p = - V_{max}\, I_{max} \sin \omega t \cos \omega t = V_{max}\, I_{max}/2 \sin 2\omega t$

The power measured by wattmeter is the average value of p which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Hence in a purely inductive circuit power absorbed is zero.

Physically the above fact can be explained as below:

During the second quarter of a cycle the current and the magnetic flux of the coil increases and the coil draws power from the supply source to build up the magnetic field (the power drawn is positive and the energy drawn by the coil from the supply source is represented by the area between the curve p and the time axis). The energy stored in the magnetic field during build up is given as $W_{max} = 1/2\, L\, I^2_{max}$.

In the next quarter the current decreases. The emf of self-induction will, however, tends to oppose its decrease. The coil acts as a generator of electrical energy, returning the stored energy in the magnetic field to the supply source (now the power drawn by the coil is negative and the curve p lies below the time axis).

The chain of events repeats itself during the next half cycles. Thus, a proportion of power is continually exchanged between the field and the inductive circuit and the power consumed by a purely inductive coil is zero.

MREC(A)

4. Purely Capacitive Circuit:

When a dc voltage is impressed across the plates of a perfect condenser, it will become charged to full voltage almost instantaneously. The charging current will flow only during the period of "build up" and will cease to flow as soon as the capacitor has attained the steady voltage of the source. This implies that for a direct current, a capacitor is a break in the circuit or an infinitely high resistance.

In Fig. 4.4 a sinusoidal voltage is applied to a capacitor. During the first quarter-cycle, the applied voltage increases to the peak value, and the capacitor is charged to that value. The current is maximum in the beginning of the cycle and becomes zero at the maximum value of the applied voltage, so there is a phase difference of 90° between the applied voltage and current. During the first quarter-cycle the current flows in the normal direction through the circuit; hence the current is positive.

In the second quarter-cycle, the voltage applied across the capacitor falls, the capacitor loses its charge, and current flows through it against the applied voltage because the capacitor discharges into the circuit. Thus, the current is negative during the second quarter-cycle and attains a maximum value when the applied voltage is zero.
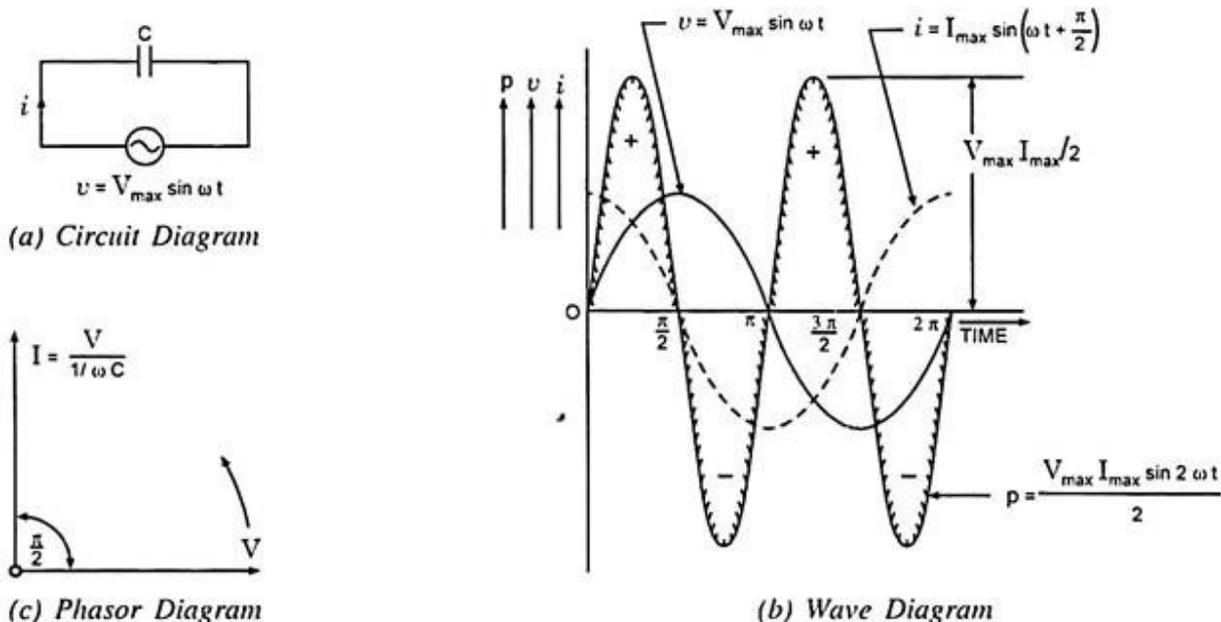
Fig. 4.4 *Purely Capacitive Circuit*

The third and fourth quarter-cycles repeat the events of the first and second, respectively, with the difference that the polarity of the applied voltage is reversed, and there are corresponding current changes.

MREC(A)

In other words, an alternating current flows in the circuit because of the charging and discharging of the capacitor. As illustrated in Figs. 4.4 (b) and (c) the current begins its cycle 90 degrees ahead of the voltage, so the current in a capacitor leads the applied voltage by 90 degrees – the opposite of the inductance current-voltage relationship.

Let an alternating voltage represented by $v = V_{max} \sin \omega t$ be applied across a capacitor of capacitance C farads.

The expression for instantaneous charge is given as:

$q = C V_{max} \sin \omega t$

Since the capacitor current is equal to the rate of change of charge, the capacitor current may be obtained by differentiating the above equation:

$$i = \frac{dq}{dt} = [C V_{max} \sin \omega t] = \omega C V_{max} \cos \omega t = \frac{V_{max}}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

Current is maximum when $t = 0$

$$\therefore \quad I_{max} = \frac{V_{max}}{1/\omega C}$$

Substituting $\dfrac{V_{max}}{1/\omega C} = I_{max}$ in the above equation for instantaneous current, we get

$$i = I_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the equations of instantaneous applied voltage and instantaneous current flowing through capacitance, it is observed that the current leads the applied voltage by π/2, as shown in Figs. 4.4 (b) and (c) by wave and phasor diagrams respectively.

Capacitive Reactance:

1/ω C in the expression $I_{max} = V_{max}/1/\omega C$ is known as capacitive reactance and is denoted by X_c i.e., XC = 1/ω C

If C is in farads and ω is in radians/s, then X_c will be in ohms.

Power in Purely Capacitive Circuit:

$$p = v i = V_{max} \sin \omega t. \ I_{max} \sin\left(\omega t + \frac{\pi}{2}\right) = V_{max} I_{max} \sin \omega t \cos \omega t$$

$$= \frac{V_{max} I_{max}}{2} \sin 2\omega t$$

Average power, $P = \dfrac{V_{max} I_{max}}{2}$ × average of sin 2 ωt over a complete cycle = 0.

Hence power absorbed in a purely capacitive circuit is zero. The same is shown graphically in Fig. 4.4 (b). The energy taken from the supply circuit is stored in the capacitor during the first quarter- cycle and returned during the next.

The energy stored by a capacitor at maximum voltage across its plates is given by the expression:

$$W_C = \frac{1}{2} C V^2_{max}$$

This can be realized when it is recalled that no heat is produced and no work is done while current is flowing through a capacitor. As a matter of fact, in commercial capacitors, there is a slight energy loss in the dielectric in addition to a minute $I^2 R$ loss due to flow of current over the plates having definite ohmic resistance.

The power curve is a sine wave of double the supply frequency. Although it raises the power factor from zero to 0.002 or even a little more, but for ordinary purposes the power factor is taken to be zero. Obviously the phase angle due to dielectric and ohmic losses decreases slightly.

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5. Resistance — Capacitance (R-C) Series Circuit:

Consider an ac circuit consisting of resistance of R ohms and capacitance of C farads connected in series, as shown in Fig. 4.18 (a).

Let the supply frequency be of fHz and current flowing through the circuit be of I amperes (rms value).
Voltage drop across resistance, $V_R = I R$ in phase with the current.
Voltage drop across capacitance, $V_C = I X_C$ lagging behind I by $\pi/2$ radians or 90°, as shown in Fig. 4.18 (b).

(a) Circuit Diagram   (b) Phasor Diagram
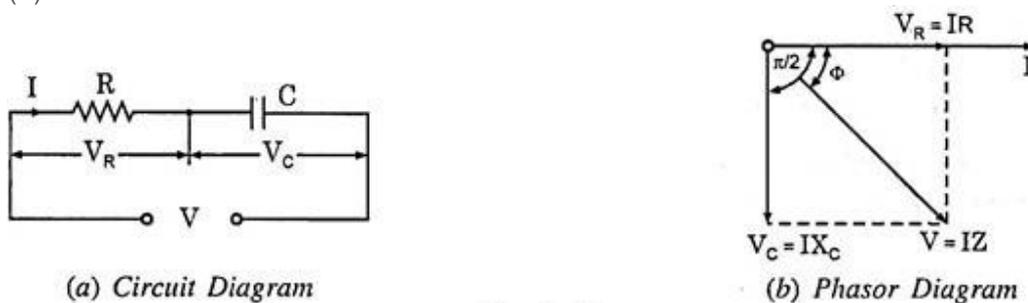
Fig. 4.18

The applied voltage, being equal to phasor sum of V_R and V_C, is given in magnitude by-

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2} = I \sqrt{R^2 + X_C^2} = IZ$$

$$\text{where } Z^2 = R^2 + X_C^2$$

The applied voltage lags behind the current by an angle φ:

MREC(A)

where $\quad \tan \Phi = \dfrac{V_C}{V_R} = \dfrac{I\,X_C}{I\,R} = \dfrac{X_C}{R} = \dfrac{1}{\omega\,R\,C}\quad$ or $\quad \Phi = \mathrm{Tan}^{-1}\ \dfrac{1}{R\,\omega\,C}$

Power factor, $\cos \Phi = \dfrac{R}{Z}$

If instantaneous voltage is represented by:

$v = V_{max} \sin \omega t$

Then instantaneous current will be expressed as:

$i = I_{max} \sin (\omega t + \phi)$

And power consumed by the circuit is given by:

$P = VI \cos \phi$

(a) Voltage Triangle   (b) Impedance Triangle
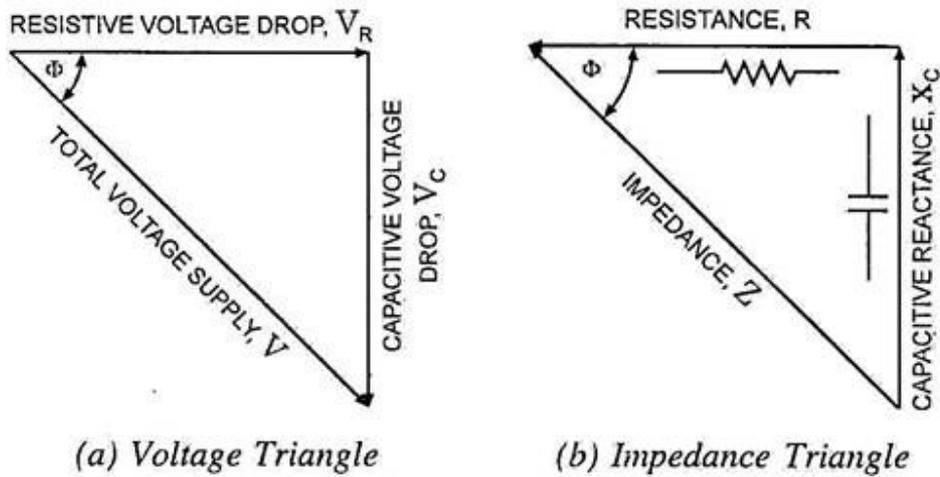
Fig. 4.19

Voltage triangle and impedance triangle Fig. 4.19 are shown in Figs. 4.19 (a) and 4.19 (b) respectively.

---

6. Apparent Power, True Power, Reactive Power and Power Factor:

The product of rms values of current and voltage, VI is called the apparent power and is measured in volt-amperes or kilo-volt amperes (kVA).

The true power in an ac circuit is obtained by multiplying the apparent power by the power factor and is expressed in watts or kilo-watts (kW).

The product of apparent power, VI and the sine of the angle between voltage and current, sin ϕ is called the reactive power. This is also known as wattless power and is expressed in reactive volt-amperes or kilo-volt amperes reactive (kVA R).

MREC(A)

i.e. Apparent power, $S = VI$ volt-amperes or $\dfrac{VI}{1,000}$ kVA

True power, $P = VI \cos \Phi$ watts or $\dfrac{VI \cos \Phi}{1,000}$ kW

Reactive power, $Q = VI \sin \Phi$ VAR or $\dfrac{VI \sin \Phi}{1,000}$ kVAR

and $kVA = \sqrt{(kW)^2 + (kVAR)^2}$

The above relations can easily be followed by referring to the power diagram shown in Fig. 4.7 (a).
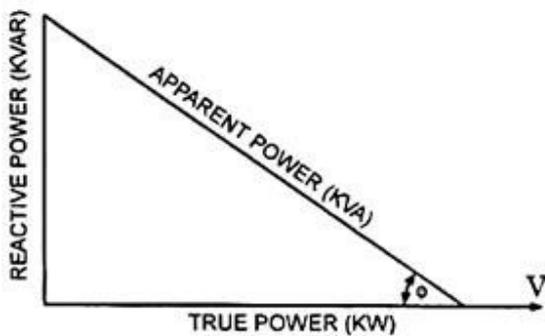
(a) *Power Triangle*
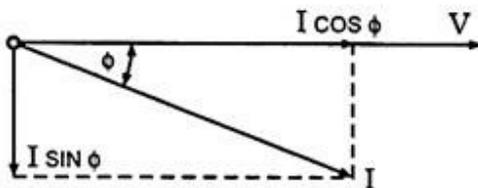
(b)

Fig. 4.7

Power factor may be defined as:

(i) Cosine of the phase angle between voltage and current,

(ii) The ratio of the resistance to impedance, or

(iii) The ratio of true power to apparent power.

The power factor can never be greater than unity. The power factor is expressed either as fraction or as a percentage. It is usual practice to attach the word 'lagging' or ' leading' with the numerical value of power factor to signify whether the current lags behind or leads the voltage.

MREC(A)

Active Component of Current:

The current component which is in phase with circuit voltage (i.e., I cos ϕ) and contributes to active or true power of the circuit is called the active (wattful or in-phase) component of current.

Reactive Component of Current:

The current component which is in quadrature (or 90° out of phase) to circuit voltage (i.e., I sin ϕ) and contributes to reactive power of the circuit, is called the reactive (or wattless) component of current.

Q-Factor of Coil:

Reciprocal of power factor is known as Q-factor of the coil. It is also called the quality factor or figure of merit of a coil.

$$\text{Mathematically Q-factor} = \frac{1}{\text{Power factor}} = \frac{1}{\text{Cos } \Phi} = \frac{Z}{R}$$

If R is very small in comparison to inductive reactance X_L, the

$$\text{Q-factor} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$\text{Also } Q = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipated per cycle}}$$

MODULE-V
Resonance& Locus Diagrams

Locus Diagrams with variation of various parameters:

Introduction: In AC electrical circuits the magnitude and phase of the current vector depends upon the values of R,L&C when the applied voltage and frequency are kept constant. The path traced by the terminus (tip) of the current vector when the parameters R,L&C are varied is called the current *Locus diagram* . Locus diagrams are useful in studying and understanding the behavior of the RLC circuits when one of these parameters is varied keeping voltage and frequency constant.

In this unit, Locus diagrams are developed and explained for series RC,RL circuits and Parallel LC circuits along with their internal resistances when the parameters R,L and C are varied.

The term circle diagram identifies locus plots that are either circular or semicircular. The defining equations of such circle diagrams are also derived in this unit for series RC and RL diagrams.

In both series RC,RL circuits and parallel LC circuits resistances are taken to be in series with L and C to highlight the fact that all practical L and C components will have at least a small value of internal resistance.

Series RL circuit with varying Resistance R:

Refer to the series RL circuit shown in the figure (a) below with constant X_L and varying R. The current I_L lags behind the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to V/X_L and lies along the I axis with phase angle equal to 90^0. When R is increased from zero to infinity the current gradually reduces from V/X_L to 0 and phase angle also reduces from 90^0 to 00. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve I axis.

MREC(A)
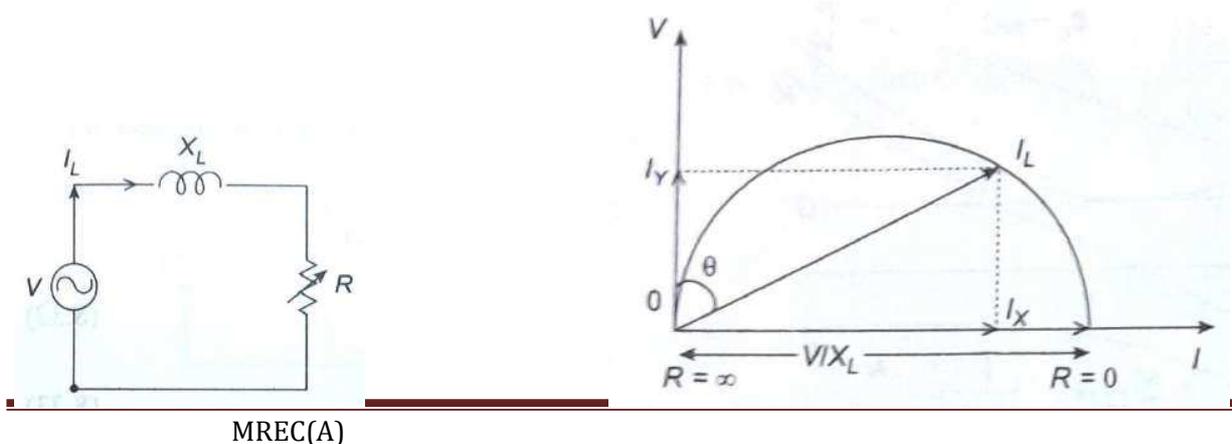
Fig 4.1(a): Series RL circuit with   **Fig 4.1(b): Locus of current vector I_L with**
variation of R

The related equations are:

I_L = V/Z   Sin Θ = X_L/Z or Z = X_L/ Sin Θ and Cos Θ = R / Z

Therefore I_L = (V/X_L) Sin Θ

For constant V and X_L the above expression for I_L is the polar equation of a circle with diameter (V/X_L) as shown in the figure above.

Circle equation for the RL circuit: (with fixed reactance and variable Resistance):

The X and Y coordinates of the current I_L are

I_X = I_L Sin Θ I_Y = I_L Cos Θ

From the relations given above and earlier we get

$$I_X = (V/Z)(X_L/Z) = V X_L/Z^2 \text{-----------------------} (1)$$

and I_Y = (V/Z)(R/Z)   = V R/Z^2 ----------------------- (2)

Squaring and adding the above two equations we get

$$I_X^2 + I_Y^2 = V^2(X_L^2 + R^2) / Z^4 = (V^2 Z^2)/ Z^4 = V^2/Z^2 \text{-------------------}(3)$$

From equation (1) above we have Z^2 = V X_L / I_X and substituting this in the above equation (3) we get :

$$I_X^2 + I_Y^2 = V^2/(V X_L / I_X) = (V/X_L) I_X \quad \text{or}$$
$$I_X^2 + I_Y^2 - (V/X_L) I_X = 0$$

Adding (V/2X_L)^2 to both sides ,the above equation can be written as

$$[I_X - V/2X_L]^2 + I_Y^2 = (V/2X_L)^2 \text{------------------------------} (4)$$

Equation (4) above represents a circle with a radius of (V/2X_L) and with it's coordinates of the centre as (V/2X_L , 0)

Series RC circuit with varying Resistance R:

Refer to the series RC circuit shown in the figure (a) below with constant X_C and varying R. The current I_C leads the applied voltage V by a phase angle Θ = $\tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to − V/X_C and lies along the negative I axis with phase angle equal to − 90^0. When R is increased from zero to infinity the current gradually reduces from −V/X_C to 0 and phase angle also reduces from −90^0 to 0^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle but now with its diameter along the negative I axis.

Circle equation for the RC circuit: (with fixed reactance and variable Resistance):

In the same way as we got for the Series RL circuit with varying resistance we can get the circle equation for an RC circuit with varying resistance as :

$$[I_X + V/2X_C]^2 + I_Y^2 = (V/2X_C)^2$$

whose coordinates of the centre are (−V/2X_C , 0) and radius equal to V/2X_C

MREC(A)
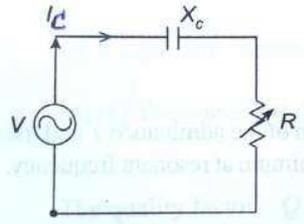
Fig 4.2 (a): Series RC circuit with
Varying Resistance R
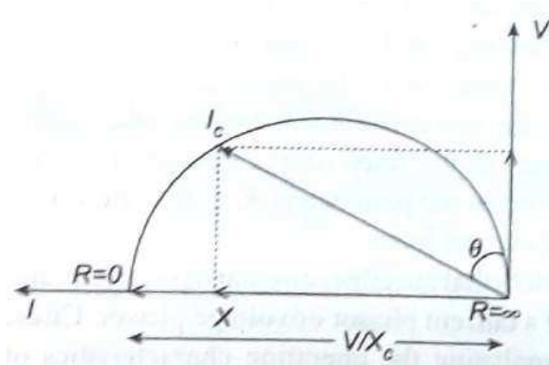
Fig 4.2 (b): Locus of current vector I_C
with variation of R

Series RL circuit with varying Reactance X_L:

Refer to the series RL circuit shown in the figure (a) below with constant R and varying X_L. The current I_L lags behind the applied voltage V by a phase angle $\theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When $X_L = 0$ we can see that the current is maximum equal to V/R and lies along the +ve V axis with phase angle equal to 0^0. When X_L is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to 90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis and on to its right side.
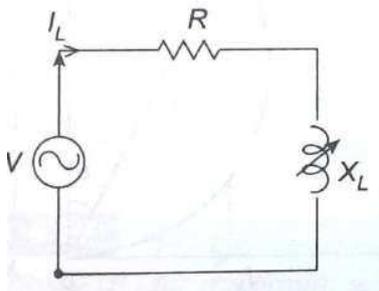
Fig 4.3(a): Series RL circuit with varying X_L Fig 4.3(b) : Locus of current vector I_L with variation of X_L
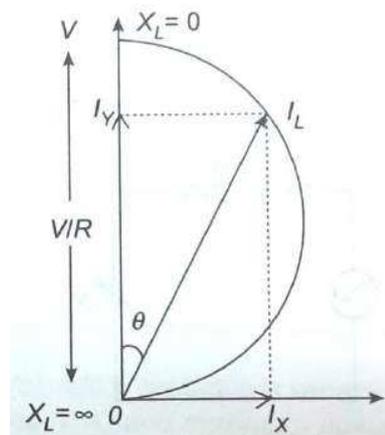
Series RC circuit with varying Reactance X_C:

Refer to the series RC circuit shown in the figure (a) below with constant R and varying X_C. The current I_C leads the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When $X_C = 0$ we can see that the current is maximum equal to V/R and lies along the V axis with phase angle equal to 0^0. When X_C is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to -90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis but now on to its left side.
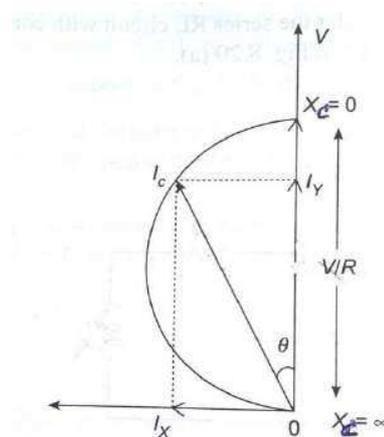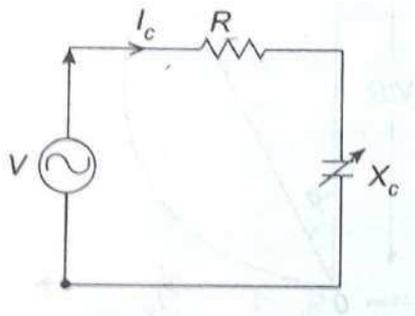
Fig 4.4(a): Series RC circuit with varying X_C Fig 4.4(b): Locus of current vector I_C with variation of X_C

Parallel LC circuits:
Parallel LC circuit along with its internal resistances as shown in the figures below is considered here for drawing the locus diagrams. As can be seen, there are two branch currents I_C and I_L along with the total current I. Locus diagrams of the current I_L or I_C (depending on which arm is varied)and the total current I are drawn by varying R_L, R_C , X_L and X_C one by one.

Varying X_L:
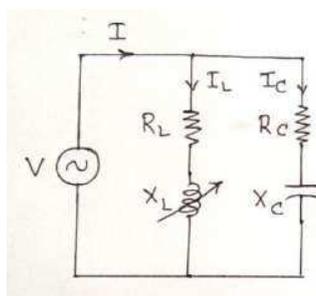
Fig 4.5(a): parallel LC circuit with Internal Resistances R_L and R_C in series with L (Variable) and C (fixed) respectively.

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L. It's amplitude is maximum and equal to V/R_L when X_L is zero and it is in phase with the applied voltage V. When X_L is increased from zero to infinity it's amplitude decreases to zero and phase will be lagging the voltage by 90^0. In between, the phase angle will be lagging the voltage V by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with X_L varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L. We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semi circle I_CIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

Fig 4.5(b): Locus of current vector I in Parallel LC circuit when X_L is varied from 0 to ∞

Varying X_C:
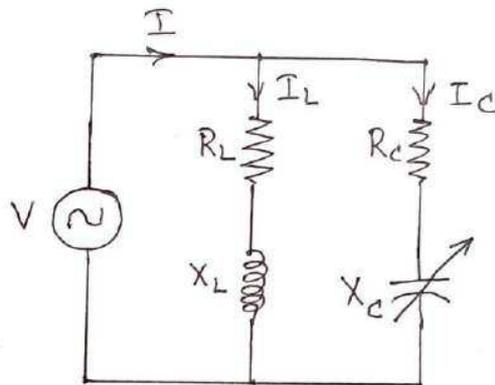
Fig.4.6(a) parallel LC circuit with Internal Resistances R_L and R_C in series with L (fixed) and C (Variable) respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C. It's amplitude is maximum and equal to V/R_C when X_C is zero and it is in phase with the applied voltage V. When X_C is increased from zero to infinity it's amplitude decreases to zero and phase will be leading the voltage by 90^0. In between, the phase angle will be leading the voltage V by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/R_C as shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with X_C varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L. We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

Fig4.6 (b) : Locus of current vector I in Parallel LC circuit when X_C is varied from 0 to ∞

Varying R_L:

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L . It's amplitude is maximum and equal to V/X_L when R_L is zero. Its phase will be lagging the voltage by 90^0. When R_L is increased from zero to infinity it's amplitude decreases to zero and it is in phase with the applied voltage V. In between, the phase angle will be lagging the voltage V by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with R varying except that here V is shown horizontally.
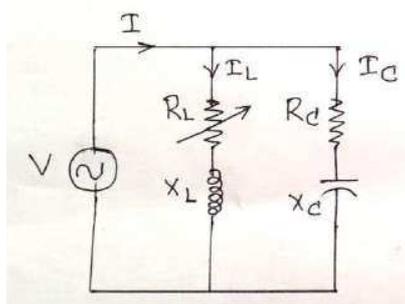
Fig. 4.7(a) parallel LC circuit with Internal Resistances R_L (Variable) and R_C (fixed) in series with L and C respectively.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semicircle I_CIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

Fig 4.7(b) : Locus of current vector I in Parallel LC circuit when R_L is varied from 0 to ∞

Varying R_C:
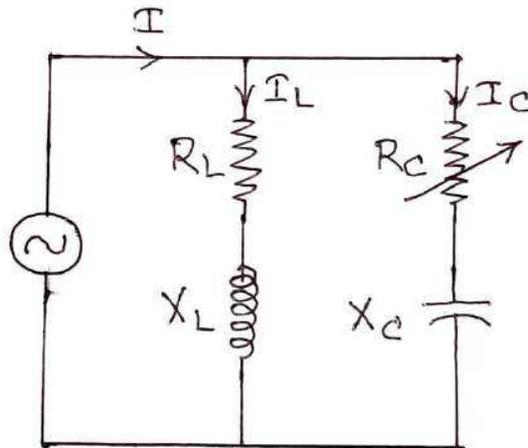
Fig. 4.8(a) parallel LC circuit with Internal Resistances R_L (fixed) and R_C (Variable)   in series with L and C respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C. It's amplitude is maximum and equal to V/X_C when R_C is zero and its phase will be leading the voltage by 90^0. When R_C is increased from zero to infinity it's amplitude decreases to zero and it will be in phase with the applied voltage V. In between, the phase angle will be leading the voltage V by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/X_C as shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with R varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L. We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

Fig 4.8(b) : Locus of current vector I in Parallel LC circuit when R_C is varied from 0 to ∞

Resonance:

Series RLC circuit:
The impedance of the series RLC circuit shown in the figure below and the current I through the circuit are given by :

$$Z = R + j\omega L + 1 / j\omega C = R + j (\omega L - 1/\omega C)$$
$$I = Vs/Z$$
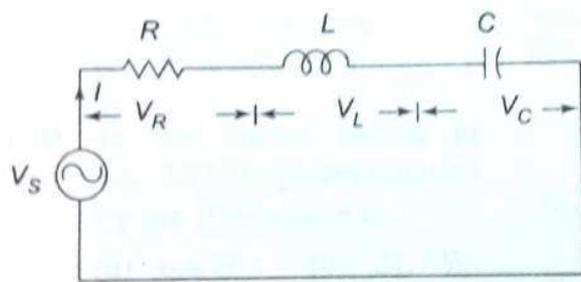
Fig 4.9: Series RLC circuit

The circuit is said to be in resonance when the Inductive reactance is equal to the Capacitive reactance. i.e. $X_L = X_C$ **or** $\omega L = 1/\omega C$. (i.e. Imaginary of the impedance is zero) The frequency at which the resonance occurs is called resonant frequency. In the resonant condition when $X_L = X_C$ they cancel with each other since they are in phase opposition(180^0 out of phase) and net impedance of the circuit is purely resistive. In this condition the magnitudes of voltages across

the Capacitance and the Inductance are also equal to each other but again since they are of opposite polarity they cancel with each other and the entire applied voltage appears across the Resistance alone.

Solving for the resonant frequency from the above condition of Resonance : **ωL = 1/ωC**

$$2\pi f_r L = 1/2\pi f_r C$$

$$f_r^2 = 1/4\pi^2 LC \quad \text{and} \quad f_r = 1/2\pi\sqrt{LC}$$

In a series RLC circuit, resonance may be produced by varying L or C at a fixed frequency or by varying frequency at fixed L and C.

Reactance, Impedance and Resistance of a Series RLC circuit as a function of frequency:

From the expressions for the Inductive and capacitive reactance we can see that when the frequency is zero, capacitance acts as an open circuit and Inductance as a short circuit. Similarly when the frequency is infinity inductance acts as an open circuit and the capacitance acts as a short circuit. The variation of Inductive and capacitive reactance along with Resistance R and the Total Impedance are shown plotted in the figure below.

As can be seen, when the frequency increases from zero to ∞ Inductive reactance X_L (directly proportional to ω) increases from zero to ∞ and capacitive reactance X_C (inversely proportional to ω) decreases from −∞ to zero. Whereas, the Impedance decreases from ∞ to Pure Resistance R as the frequency increases from zero to f_r (as capacitive reactance reduces from −∞ and becomes equal to Inductive reactance) and then increases from R to ∞ as the frequency increases from f_r to ∞ (as inductive reactance increases from its value at resonant frequency to ∞)
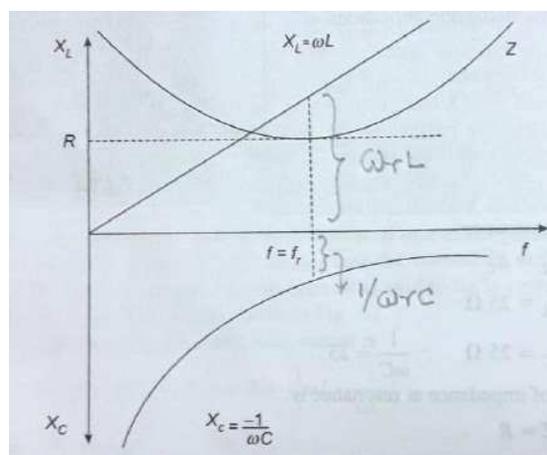
Fig 4.10: Reactance and Impedance plots of a Series RLC circuit

Phase angle of a Series RLC circuit as a function of frequency:
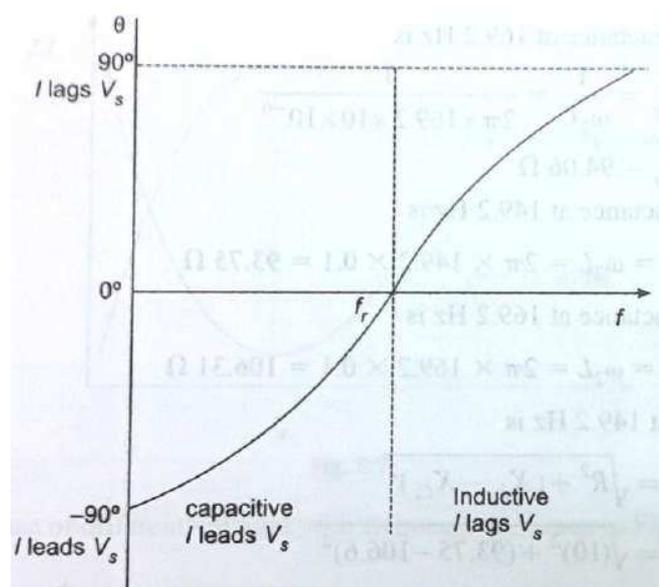
Fig4.11 : Phase plot of a Series RLC circuit

The following points can be seen from the Phase angle plot shown in the figure above:

- At frequencies below the resonant frequency capacitive reactance is higher than the inductive reactance and hence the phase angle of the current leads the voltage.
- As frequency increases from zero to f_r the phase angle changes from -90^0 to zero.
- At frequencies above the resonant frequency inductive reactance is higher than the capacitive reactance and hence the phase angle of the current lags the voltage.
- As frequency increases from f_r and approaches ∞, the phase angle increases from zero and approaches 90^0

Band width of a Series RLC circuit:
The band width of a circuit is defined as the Range of frequencies between which the output power is half of or 3 db less than the output power at the resonant frequency. These frequencies are called the cutoff frequencies, 3db points or half power points. But when we consider the output voltage or current, the range of frequencies between which the output voltage or current falls to 0.707 times of the value at the resonant frequency is called the Bandwidth BW. This is because voltage/current are related to power by a factor of $\sqrt{2}$ and when we are consider $\sqrt{2}$ times less it becomes 0.707. But still these frequencies are called as cutoff frequencies, 3db points or half power points. The lower end frequency is called *lower cutoff frequency* and the higher end frequency is called upper *cutoff frequency.*
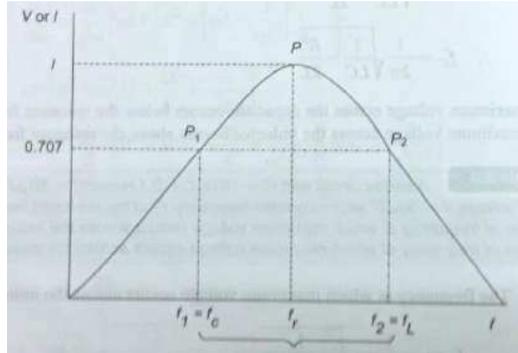
Fig 4.12: Plot showing the cutoff frequencies and Bandwidth of a series RLC circuit

Derivation of an expression for the BW of a series RLC circuit:

We know that BW = $f_2 - f_1$ Hz

If the current at points P_1 and P_2 are 0.707 ($1/\sqrt{2}$) times that of I_{max} (current at the resonant frequency) then the Impedance of the circuit at points P_1 and P_2 is $\sqrt{2}$ R (i.e. $\sqrt{2}$ times the impedance at f_r)

But Impedance at point P_1 is given by: $Z = \sqrt{R^2 + (1/\omega_1 C - \omega_1 L)^2}$ and equating this to $\sqrt{2}$ R we get : $\qquad (1/\omega_1 C) - \omega_1 L = R \qquad$ ------ $\qquad$ (1)

Similarly Impedance at point P_2 is given by: $Z = \sqrt{R^2 + (\omega_2 L - 1/\omega_2 C)^2}$ and equating this to $\sqrt{2}$ R we get: $\qquad \omega_2 L - (1/\omega_2 C) = R \qquad$ ------ $\qquad$ (2)

Equating the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L = \omega_2 L - 1/\omega_2 C$$

Rearranging we get $\qquad L(\omega_1 + \omega_2) = 1/C [(\omega_1 + \omega_2)/\omega_1 \omega_2]$ $\quad$ i.e $\qquad \omega_1 \omega_2 = 1/LC$

But we already know that for a series RLC circuit the resonant frequency is given by $\omega^2 = 1/LC$

Therefore: $\qquad \omega_1 \omega_2 = \omega_r^2$ ---- (3) $\qquad$ and $\qquad 1/C = \omega_r^2 L \qquad$ ------ $\qquad$ (4)

Next adding the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L + \omega_2 L - 1/\omega_2 C = 2R$$

$$(\omega_2 - \omega_1)L + (1/\omega_1 C - 1/\omega_2 C) = 2R$$

$$(\omega_2 - \omega_1)L + 1/C[(\omega_2 - \omega_1)/\omega_1 \omega_2] = 2R \qquad ------ \qquad (5)$$

Using the values of $\omega_1 \omega_2$ and 1/C from equations (3) and (4) above into equation (5) above we get: $\qquad (\omega_2 - \omega_1)L + \omega_r^2 L [(\omega_2 - \omega_1)/\omega_r^2] = 2R$

i.e. $2L(\omega_2 - \omega_1) = 2R$ i.e. $(\omega_2 - \omega_1) = R/L$ and $(f_2 - f_1) = R/2\pi L$ ----------- (6)

Or finally Band width   **BW = R/2πL** -- (7)

Since f_r lies in the centre of the lower and upper cutoff frequencies f_1 and f_2 using the above equation (6) we can get:

$$f_1 = f_r - R/4\pi L \qquad\qquad \text{------ (8)}$$

$$f_2 = f_r + R/4\pi L \qquad\qquad \text{------ (9)}$$

Further by dividing the equation (6) above by f_r on both sides we get another **important** relation : $(f_2 - f_1) / f_r = R/2\pi f_r L$ or $BW / f_r = R/2\pi f_r L$ ----------------- (10)

Here an important property of a coil i.e. **Q factor** or **figure of merit** is defined as the ratio of the reactance to the resistance of a coil.

$$Q = 2\pi f_r L / R \text{ --------------------------------(11)}$$

Now using the relation (11) we can rewrite the relation (10) as

$$Q = f_r / BW \text{ ---------------------------------- (12)}$$

Quality factor of a series RLC circuit:

The quality factor of a series RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance / Average power at Resonance

Reactive power in Inductor at resonance = $I^2 X_L$

Reactive power in Capacitor at resonance = $I^2 X_C$

Average power at Resonance   = $I^2 R$

Here the power is expressed in the form $I^2 X$ (not as V^2/X) since I is common through R.L and C in the series RLC circuit and it gets cancelled during the simplification.

Therefore **Q = $I^2 X_L$ / $I^2 R$ = $I^2 X_C$ / $I^2 R$**

i.e.   **Q = X_L / R = ω_r L/ R** -------------------------------------- (1)

Or   **Q = X_C / R = 1/ω_r RC** --------------------------------------(2)

From these two relations we can also define Q factor as :

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Q = Inductive (or Capacitive) reactance at resonance / Resistance

Substituting the value of $\omega_r = 1/\sqrt{LC}$ in the expressions (1) or (2) for **Q** above we can get the value of **Q** in terms of **R, L,C** as below.

$$Q = (1/\sqrt{LC})\, L/R = (1/R)(\sqrt{L/C})$$

Selectivity:

Selectivity of a series **RLC** circuit indicates how well the given circuit responds to a given resonant frequency and how well it rejects all other frequencies. i.e. the selectivity is directly proportional to **Q** factor. A circuit with a good selectivity (or a high **Q** factor) will have maximum gain at the resonant frequency and will have minimum gain at other frequencies .i.e. it will have very low band width. This is illustrated in the figure below.
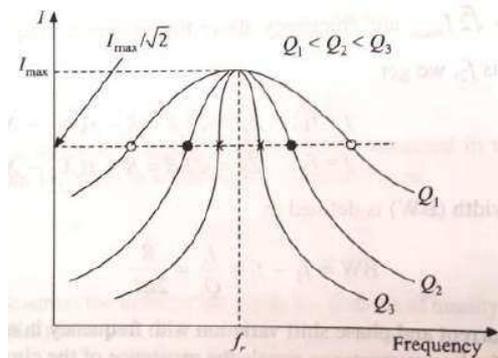
Fig 4.13: Effect of quality factor on bandwidth Voltage Magnification at resonance:

At resonance the voltages across the Inductance and capacitance are much larger than the applied voltage in a series RLC circuit and this is called voltage magnification at Resonance. The voltage magnification is equal to the **Q** factor of the circuit. This is proven below.
If we take the voltage applied to the circuit as **V** and the current through the circuit at resonance as **I** then
The voltage across the inductance **L** is: $V_L = IX_L = (V/R)\,\omega_r\,L$ and
The voltage across the capacitance **C** is: $V_C = IX_C = V/R\,\omega_r\,C$
But we know that the **Q** of a series RLC circuit = $\omega_r\,L/R = 1/R\,\omega_r\,C$
Using these relations in the expressions for **V_L** and **V_C** given above we get
$V_L = VQ$ and $V_C = VQ$
The ratio of voltage across the Inductor or capacitor at resonance to the applied voltage in a series RLC circuit is called Voltage magnification and is given by
$$\text{Magnification} = Q = V_L/V \quad \text{or} \quad V_C / V$$

Important points In Series RLC circuit at resonant frequency :

- The impedance of the circuit becomes purely resistive and minimum i.e **Z = R**
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is **Q** times higher than the voltage across the resistor

Bandwidth and Q factor of a Parallel RLC circuit:

Parallel RLC circuit is shown in the figure below. For finding out the **BW** and **Q** factor of a parallel RLC circuit, since it is easier we will work with Admittance , Conductance and Susceptance instead of Impedance ,Resistance and Reactance like in series RLC circuit.
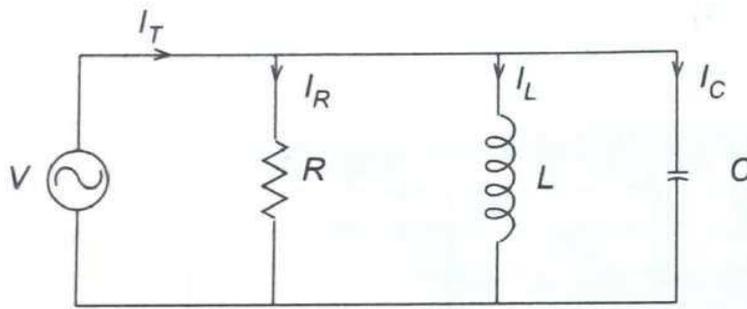
Fig 4.14 : Parallel RLC circuit

Then we have the relation: $Y = 1/Z = 1/R + 1/j\omega L + j\omega C = 1/R + j(\omega C - 1/\omega L)$

For the parallel RLC circuit also, at resonance, the imaginary part of the Admittance is zero and hence the frequency at which resonance occurs is given by: $\omega_r C - 1/\omega_r L = 0$. From this we get : $\omega_r C = 1/\omega_r L$ and $\omega_r = 1/\sqrt{LC}$
which is the same value for ω_r as what we got for the series RLC circuit.

At resonance when the imaginary part of the admittance is zero the *admittance* becomes *minimum*.(i.e *Impedance* becomes *maximum* as against Impedance becoming minimum in series RLC circuit) i.e. Current becomes minimum in the parallel RLC circuit at resonance (as against current becoming maximum in series RLC circuit) and increases on either side of the resonant frequency as shown in the figure below.
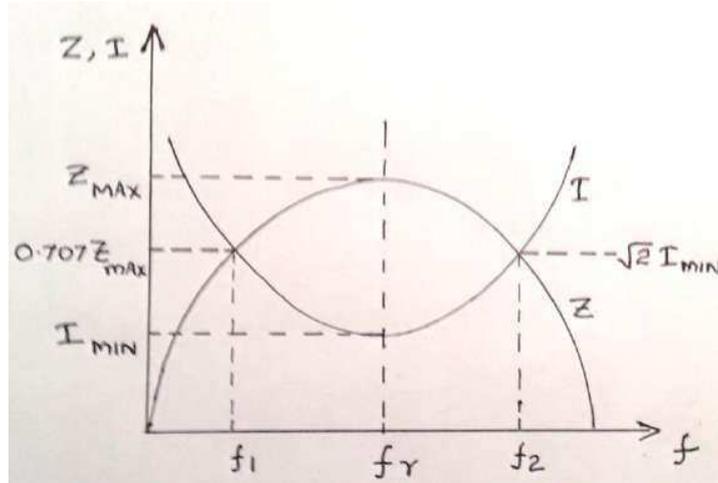
Fig 4.15: Variation of Impedance and Current with frequency in a Parallel RLC circuit

Here also the BW of the circuit is given by BW = f_2-f_1 where f_2 and f_1 are still called the upper and lower cut off frequencies but they are 3db higher cutoff frequencies since we notice that at these cutoff frequencies the amplitude of the current is $\sqrt{2}$ times higher than that of the amplitude of current at the resonant frequency.

The BW is computed here also on the same lines as we did for the series RLC circuit:

If the current at points P_1 and P_2 is $\sqrt{2}$ (3db) times higher than that of I_{min}(current at the resonant frequency) then the admittance of the circuit at points P_1 and P_2 is also $\sqrt{2}$ times higher than the admittance at f_r)

But amplitude of admittance at point P_1 is given by: $Y = \sqrt{1/R^2 + (1/\omega_1 L - \omega_1 C)^2}$ and equating this to $\sqrt{2}/R$ we get

$1/\omega_1 L - \omega_1 C \quad = \quad 1/R$ ---------------- (1)

Similarly amplitude of admittance at point P_2 is given by: $Y = \sqrt{1/R^2 + (\omega_2 C - 1/\omega_2 L)^2}$ and equating this to $\sqrt{2}/R$ we get

$\omega_2 C - 1/\omega_2 L \quad = \quad 1/R$ ---------------- (2)

Equating LHS of (1) and (2) and further simplifying we get

$$1/\omega_1 L - \omega_1 C \quad = \quad \omega_2 C - 1/\omega_2 L$$

$$1/\omega_1 L + 1/\omega_2 L \quad = \quad \omega_1 C + \omega_2 C$$

$$1/L \, [(\omega_1 + \omega_2)/ \, \omega_1 \omega_2] = (\omega_1 + \omega_2)C$$

$$1/L \, C \quad = \quad \omega_1 \omega_2$$

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Next adding the equations (1) and (2) above and further simplifying we get

$$1/\omega_1 L - \omega_1 C + \omega_2 C - 1/\omega_2 L = 2/R$$

$$(\omega_2 C - \omega_1 C) + (1/\omega_1 L - 1/\omega_2 L) = 2/R$$

$$(\omega_2 - \omega_1)C + 1/L\ [(\omega_2 - \omega_1)/\ \omega_1 \omega_2] = 2/R$$

Substituting the value of $\omega_1 \omega_2 = 1/LC$

$$(\omega_2 - \omega_1)C + LC/L\ [(\omega_2 - \omega_1)] = 2/R$$

$$(\omega_2 - \omega_1)C + C\ [(\omega_2 - \omega_1)] = 2/R$$

$$2\,C\ [(\omega_2 - \omega_1)] = 2/R$$

$$\text{Or } [(\omega_2 - \omega_1)] = 1/RC$$

From which we get the band width **BW = f_2-f_1 = 1/2π RC**

Dividing both sides by f_r we get :   **(f_2-f_1)/ f_r = 1/2π f_r RC----------(1)**

Quality factor of a Parallel RLC circuit:

The quality factor of a Parallel RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance / Average power at Resonance

Reactive power in Inductor at resonance = V^2/X_L

Reactive power in Capacitor at resonance = V^2/X_C

Average power at Resonance   = V^2/R

Here the power is expressed in the form V^2/X (not as I^2X as in series circuit) since V is common across R,L and C in the parallel RLC circuit and it gets cancelled during the simplification.

Therefore **Q = (V^2/X_L) / (V^2/R) = (V^2/X_C) / (V^2/R)**

i.e. **Q = R/ X_L = R /ω_r L**---(1)

Or **Q = R/ X_C = ω_r RC** ---(2)

From these two relations we can also define **Q** factor as :

Q = Resistance /Inductive (or Capacitive) reactance at resonance

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Substituting the value of $\omega_r = 1/\sqrt{LC}$ in the expressions (1) or (2) for **Q** above we can get the value of **Q** in terms of R, L,C as below.

$$Q = (1/\sqrt{LC})\, RC = R\,(\sqrt{C/L})$$

Further using the relation $Q = \omega_r RC$ (equation 2 above) in the earlier equation (1) we got in BW viz. $(f_2-f_1)/f_r = 1/2\pi f_r RC$ we get : $(f_2-f_1)/f_r = 1/Q$ or $Q = f_r/(f_2-f_1) = f_r/BW$

i.e. In Parallel RLC circuit also the Q factor is inversely proportional to the BW.

Admittance, Conductance and Susceptance curves for a Parallel RLC circuit as a function of frequency :

- The effect of varying the frequency on the Admittance, Conductance and Susceptance of a parallel circuit is shown in the figure below.
- Inductive susceptance B_L is given by $B_L = -1/\omega L$. It is inversely proportional to the frequency ω and is shown in the in the fourth quadrant since it is negative.
- Capacitive susceptance B_C is given by $B_C = \omega C$. It is directly proportional to the frequency ω and is shown in the in the first quadrant as OP .It is positive and linear.
- Net susceptance $B = B_C - B_L$ and is represented by the curve **JK**. As can be seen it is zero at the resonant frequency f_r
- The conductance $G = 1/R$ and is constant
- The total admittance **Y** and the total current **I** are minimum at the resonant frequency as shown by the curve **VW**
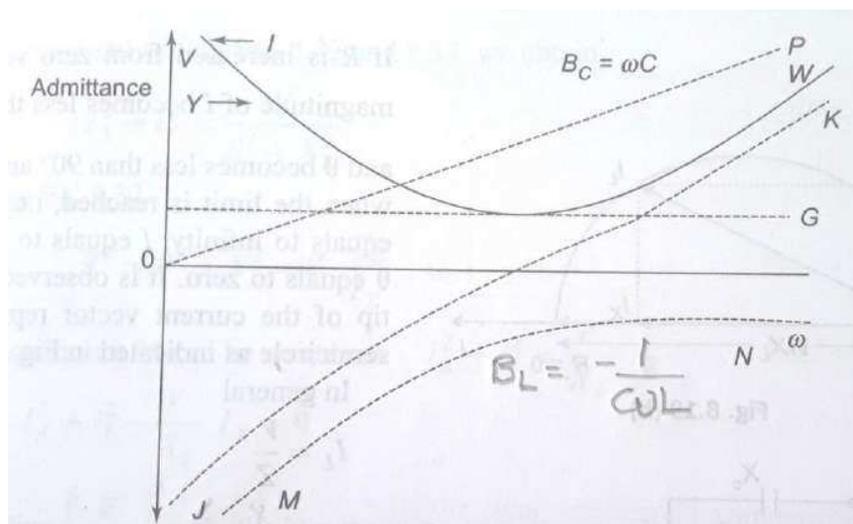
Fig 4.16: Conductance, Susceptance and Admittance plots of a Parallel RLC circuit

Current magnification in a Parallel RLC circuit:

Just as voltage magnification takes place across the capacitance and Inductance at the resonant frequency in a series RLC circuit, current magnification takes place in the currents through the capacitance and Inductance at the resonant frequency in a Parallel RLC circuit. This is shown below.

Voltage across the Resistance **= V = IR**

Current through the Inductance at resonance I_L **= V/ ω_r L = IR / ω_r L = I . R/ ω_r L = I Q**

Similarly

Current through the Capacitance at resonance I_C **= V/ (1/ω_r C) = IR / (1/ω_r C) = I(R ω_r C) = I Q**

From which we notice that the quality factor **Q = I_L / I or I_C / I** and that the current through the inductance and the capacitance increases by **Q** times that of the current through the resistor at resonance. .

Important points In Parallel RLC circuit at resonant frequency :

- The impedance of the circuit becomes resistive and maximum i.e **Z = R**
- The current in the circuit becomes minimum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The current through the Capacitor becomes equal and opposite to the current through the Inductor at resonance and is Q times higher than the current through the resistor